The Principle of Linear Superposition

1 – Conditions for Interference

When two or more light waves meet at a given point, their electric fields combine (interfere) according to the laws of linear superposition:

- The waves can add together either constructively or destructively.
- In **constructive interference**, the amplitude of the resultant wave is greater than that of the individual waves.
- In **destructive interference**, the amplitude of the resultant wave is less than that of the individual waves.
- Interference effects between two or more light sources are observable,
  1. if the sources are **coherent**, *ie* they must maintain a constant phase with respect to each other, and
  2. if the sources produce light of identical wavelengths.

- If a single wavelength source is used to illuminate a screen with two small slits, the light emerging from the two slits is coherent.

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2 – Young’s Double-Slit Experiment

- Interference in light waves from two sources was first demonstrated in 1801 by Thomas Young:
  
  The light from the two slits produces on a screen a visible pattern of a series of bright and dark parallel bands called **fringes**.

- The bright (dark) bands are due to constructive (destructive) interference.
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- Quantitative description:

A wave from the lower slit travels farther than a wave from the upper slit by an amount $\Delta l = d \sin \theta$, where $d$ is the separation of the two slits. This equation assumes that the distance of the screen to the slits, $L$, is $L \gg d$.

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- **Constructive interference** occurs, if the path difference is an integral multiple of the wavelength:

$$\Delta l = d \sin \theta = m\lambda$$

(1)

with $m = 0, \pm 1, \pm 2, \ldots$ $m$ is called the **order number** of the fringe. The central fringe is called the **zeroth-order maximum**.

- When the path difference is an odd multiple of $\lambda/2$, the two waves arriving at the screen are $180^\circ$ out of phase $\rightarrow$ destructive interference. The condition for **destructive interference** is:

$$\Delta l = d \sin \theta = \left(m + \frac{1}{2}\right)\lambda$$

(2)

with $m = 0, \pm 1, \pm 2, \ldots$
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The distance between the bright fringes on a screen measured from the zeroth order maximum, \( y \), is given by (for \( L \gg d \gg \lambda \)):

\[
\sin \theta \approx \tan \theta \approx \frac{y}{L}
\]

and therefore the position of the bright fringes on the screen is given by

\[
y_{\text{bright}} = \frac{\lambda L}{d} m
\]

Similarly, the position of the dark fringes is

\[
y_{\text{dark}} = \frac{\lambda L}{d} \left( m + \frac{1}{2} \right)
\]

3 – Interference in Thin Films

Interference effects are commonly observed in thin films (e.g., soap bubbles, oil on water). The varied colors observed with ordinary white light result from the interference of waves reflected from the opposite surfaces of the film.

Consider a film of uniform thickness \( t \) and index of refraction \( n \):
1. An electromagnetic wave traveling from a medium with index of refraction $n_1$ toward one with index of refraction $n_2$ undergoes a $180^\circ$ phase change upon reflection if $n_2 > n_1$.

2. There is no phase change if $n_1 > n_2$.

3. The wavelength of light, $\lambda_n$, in a medium with index of refraction $n$ is

$$\lambda_n = \frac{\lambda_{\text{vacuum}}}{n}$$

where $\lambda_{\text{vacuum}}$ is the wavelength in vacuum.

- A $180^\circ$ phase shift is equivalent to a path difference of $\lambda_n/2$ between ray 1 and ray 2.
- Ray 2 travels an extra distance of $2t$ before it recombines with ray 1.

In general the condition for constructive interference then is

$$2t = \left( m + \frac{1}{2} \right) \lambda_n, \quad (m = 0, 1, 2, \ldots)$$

or

$$2nt = \left( m + \frac{1}{2} \right) \lambda_{\text{vacuum}}, \quad (m = 0, 1, 2, \ldots)$$

- The condition for destructive interference is

$$2nt = m\lambda_{\text{vacuum}}, \quad (m = 0, 1, 2, \ldots)$$

- These equations are only correct, if the medium above and below the film is the same. If the medium below the film has a higher index of refraction than the film, there is a $180^\circ$ phase shift for both rays, and the conditions for constructive and destructive interference are reversed.
4 – The Michelson Interferometer

- The Michelson interferometer is a device that splits a light beam into two parts and then recombines them to form an interference pattern.
- It is used to make accurate length measurements.
- The interference pattern for the two rays is determined by the difference in their path lengths, \( D_F - D_A = m\lambda \) or \((m + 1/2)\lambda\).

5 – Diffraction

- Suppose a light beam is incident on a narrow slit. If the light traveled in straight lines after passing through the slit, only the region directly opposite to the slit would be illuminated.
- Instead, Huygens’ principle requires that the wave spreads out. In other words, the light deviates from a straight line path and enters a region that would otherwise be shadowed.
- The divergence of light from its initial line of travel is called **diffraction**.
- In general diffraction occurs when waves pass through small openings, by sharp edges etc, i.e. when light gets bend around obstacles.
- So-called **Fraunhofer diffraction** occurs when the rays reaching the observing screen are approximately parallel. This can be achieved by placing the screen far from the slit, or by using a converging lens to focus the rays. Otherwise it is called **Fresnel diffraction**.
So far we have assumed that slits are point sources of light. What happens when the finite width of a slit is taken into account?

Huygens’ principle implies that each portion of the slit acts as a source of waves.

Hence, light from one portion of the slit can interfere with light from another portion → diffraction occurs.

Quantitative analysis: Consider an incident wave front and identify several points that are sources for a light wavelet. All waves originating at the slit are in phase.

Focus on wave 1 and wave 3. The difference in their paths is \( W/2 \sin \theta \), where \( W \) is the width of the slit. If this path difference equals \( \lambda/2 \), the two waves cancel each other, ie destructive interference occurs. The condition for destructive interference is

\[
\frac{W}{2} \sin \theta = \frac{\lambda}{2} \Rightarrow \sin \theta = \frac{\lambda}{W}
\]
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- If we divide the slit into 4 parts instead of two, we find that destructive interference also occurs for \( \sin \theta = \frac{2\lambda}{W} \).

The general condition for **destructive interference** is

\[
\sin \theta = m \frac{\lambda}{W} \tag{5}
\]

with \( m = \pm 1, \pm 2, \pm 3, \ldots \).

- General intensity distribution along the screen:

\begin{center}
\includegraphics{intensity_distribution.png}
\end{center}

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7 – Resolution of Single-Slit and Circular Apertures

- The ability of an optical system (e.g., microscope, telescope) to distinguish between closely spaced objects is limited because of the wave nature of light.

\begin{center}
\includegraphics{resolution.png}
\end{center}

- Without diffraction, one would observe two distinct bright spots.
- Because of diffraction, each source appears as a bright central region flanked by weaker dark and bright rings.
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- If the two sources are separated so that their central maxima do not overlap, their images can be distinguished and are said to be resolved. Otherwise the images are not resolved.

- Criterion to decide whether two images are resolved (Rayleigh’s criterion): When the central maximum of one image falls on the first minimum of the 2nd image, the images are said to be just resolved.

- From Rayleigh’s criterion one can determine the minimum angular separation, \( \theta_{min} \), subtended by the source at the opening so that the images will be just resolved. The location of the first minimum in a circular opening diffraction pattern depends on the diameter of the opening, \( D \):

\[
\sin \theta = 1.22 \frac{\lambda}{D}
\]

In most situations, \( \lambda \ll D \) and therefore \( \sin \theta \approx \theta \). Thus,

\[
\theta_{min} \approx 1.22 \frac{\lambda}{D} \quad ; \quad \theta_{min} \text{ in radians}
\]

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- The minimum separation, \( y \), between two objects that can just be resolved when viewed through a defracting opening is

\[
y = L \theta_{min} \quad \text{with} \quad \theta_{min} \text{ in radians}
\]

where \( L \) is the distance between the objects and the opening.

- The minimum angular separation, \( \theta_{min} \), subtended by the source at a slit is determined by the first minimum in a single slit diffraction pattern:

\[
\sin \theta = \frac{\lambda}{W} \quad \Rightarrow \quad \theta_{min} \approx \frac{\lambda}{W}
\]

where \( W \) is the width of the slit.