The Coulomb force law is of the same form as the universal law of gravity → the electrostatic force is a conservative force.

**Example:** A small positive test charge, \( q_0 \), in a uniform electric field \( E \):

\[
W_{AB} = F \cdot s = mg(h_A - h_B)
\]

Work done by constant elec. force:

\[
W_{AB} = F \cdot s = |q_0|E \cdot s
\]

Work=Change in \( PE \):

\[
W_{AB} = -\Delta GPE = GPE_A - GPE_B
\]

\[
W_{AB} = -\Delta EPE = EPE_A - EPE_B
\]

- A positive charge gains electric potential energy when it is moved in a direction opposite the electric field.

- A negative charge loses electric potential energy when it is moved in a direction opposite the electric field.

**Energy Conservation:** \( W_{nc} = 0J \Rightarrow \Delta E = E_f - E_0 = 0J \)

The total energy, \( E \), of the universe does not change. Energy can only be transformed between different forms of energy but can never be destroyed or created out of nothing.

\[
E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh + \frac{1}{2}kx^2 + EPE
\]
**Electric Potential:**

The potential difference between points A and B, $\Delta V = V_B - V_A$, is defined as the change in electric potential energy of a charge, $q_0$, moved from A to B, divided by the charge.

$$\Delta V = V_B - V_A = \frac{\Delta EPE}{q_0} = \frac{-W_{AB}}{q_0}$$  \hspace{1cm} (2)

SI Unit: Volt: $1\, V = 1\, J/C$ ($1\, eV = 1.6 \times 10^{-19}\, J$)

Note: Only potential differences can be measured, not the absolute values.

**2 – Electric Potential due to a Point Charge**

It is possible to define the potential of a point charge at a point in space. Reference point: Point of zero potential for a point charge: $\infty$

*Electric potential due to a point charge, $q$, at a distance $r$ from the charge:*

$$V = k\frac{q}{r}$$  \hspace{1cm} (3)

- $V$ only depends on $q$ and $r$.

- The electric potential of two or more charges is obtained by applying the **superposition principle**: The total potential at a point $P$ is the sum of the potentials due to the individual point charges present.
q > 0 → V > 0, i.e. the potential has been raised with respect to the zero reference value.
q < 0 → V < 0, i.e. the potential has decreased with respect to the zero reference value.

**Electric Potential Energy of two point charges** $q_1$ and $q_2$:

1. First, the charge $q_1$ is moved from $\infty$ to point A. Since there is no other charge present, no electric work is done and thus $EPE_{A1} = 0$.
2. Then, the charge $q_2$ is moved from $\infty$ to point B which is a distance $r$ apart from A. Now electric work must be done due to the electric potential created by charge $q_1$: $W_{\infty B} = -q_2 V_1$ and thus $EPE_{B2} = q_2 V_1$.

\[
EPE = EPE_{A1} + EPE_{B2} = 0 + q_2 V_1 = q_1 V_2 + 0 = k \frac{q_1 q_2}{r} \quad (4)
\]

- If the two charges have the same sign, $EPE$ is positive (like charges repel, so positive work must be done to bring the two charges near one another)
Electrical Potential Energy and Electric Potential

3 – Potentials and Charged Conductors

Relation between electric work and electric potential when a charge $q_0$ is moved from point A to B:

$$W_{AB} = -\Delta EPE, \quad \Delta EPE = q_0(V_B - V_A) \rightarrow$$

$$W_{AB} = -q_0(V_B - V_A) \quad (5)$$

- **No work** ($W = 0$) **is required to move a charge between two points that are at the same potential** ($V_B = V_A$)

  On a charged conductor, $\mathbf{E}$ is perpendicular to its surface, and ZERO inside $\rightarrow$ no work is done if a charge is moved $\rightarrow$

- **The electric potential is a constant everywhere on the surface of a charged conductor**

  Furthermore, it is constant everywhere inside a conductor, and equal to its value at the surface.

---

**An area on which all points are at the same potential is called an equipotential surface.**

- The electric field $\mathbf{E}$ at every point of an equipotential surface is perpendicular to the surface.
4 – Capacitors and Dielectrics

A capacitor is a device to store electric charge (electric energy) and is used in a variety of electric circuits.

A capacitor consists of two conductors placed near to each other without touching:

Definition of capacitance:

\[ C = \frac{q}{V} \]  

SI unit: 1 F = 1 C/V (Farad)

5 – The Parallel Plate Capacitor

The capacitance of a device depends on the geometric arrangement of the conductors. For a parallel plate capacitor whose plates are separated by a vacuum:

\[ C' = \varepsilon_0 \frac{A}{d} \]  

\( \varepsilon_0 \) is the permittivity of free space: \( \varepsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2 \)

If an insulator (dielectric) is placed between the two conductors, the electric field between the conductors decreases by \((\kappa=\text{dielectric constant, see Tab.19.1})\)

\[ E = \frac{E_0}{\kappa} \]  

and the capacitance increases, i.e. more charge can be stored per volt.

For a parallel plate capacitor whose plates are separated by a dielectric:

\[ C' = \kappa \varepsilon_0 \frac{A}{d} \]
6 – Energy stored in a Charged Capacitor

1. Start with an uncharged capacitor and move a small charge, $\Delta q$ on capacitor. Voltage: $\delta V = \delta q / C$

2. As the charge accumulates and the voltage increases: to move a charge $\delta q$ on capacitor which is at a voltage $V$, work, $\delta W = V \delta q$, needs to be done.

3. The total work done is
$$ W = \frac{1}{2} q V $$

4. The work done is stored as energy:
$$ \text{Energy stored} = \frac{1}{2} q V = \frac{1}{2} CV^2 = \frac{q^2}{2C} \quad (11) $$

Parallel plate capacitor (with $V = Ed$):
$$ \text{Energy stored} = \frac{1}{2} CV^2 = \frac{1}{2} C(Ed)^2 \quad (12) $$