4.4 The Lagrangian of the GWS model

Finally, here is the complete Lagrangian of the GWS model, which is part of the Standard Model Lagrangian describing leptons and their electroweak interaction (in unitary gauge, i.e. no would-be Goldstone bosons occur):

$$\mathcal{L}_{GWS} = \sum_f (\bar{\Psi}_f(i\gamma^\mu \partial\mu - m_f)\Psi_f - eQ_f\bar{\Psi}_f\gamma^\mu\Psi_f A_\mu)+$$

$$+\frac{g}{\sqrt{2}}\sum_i(\bar{a}^i_L\gamma^\mu b^i_L W^+_\mu + \bar{b}^i_L\gamma^\mu a^i_L W^-_\mu) + \frac{g}{2c_w}\sum_f \bar{\Psi}_f\gamma^\mu(I^3_f - 2s^2_w Q_f - I^3_f\gamma_5)\Psi_f Z_\mu +$$

$$-\frac{1}{4}|\partial_\mu A_\nu - \partial_\nu A_\mu - ie(W^-_\mu W^+_\nu - W^+_\mu W^-_\nu)|^2 - \frac{1}{2}|\partial_\mu W^+_\nu - \partial_\nu W^+_\mu +$$

$$-ie(W^+_\mu A_\nu - W^+_\nu A_\mu) + ig' c_w (W^+_\mu Z_\nu - W^+_\nu Z_\mu|^2 +$$

$$-\frac{1}{4}|\partial_\mu Z_\nu - \partial_\nu Z_\mu + ig' c_w(W^-_\mu W^+_\nu - W^+_\mu W^-_\nu)|^2 +$$

$$-\frac{1}{2}M^2_\eta \eta^2 - \frac{gM^2_\eta}{8M_W}\eta^3 - \frac{g'^2 M^2_\eta}{32 M_W}\eta^4 + |M_W W^+_\mu + \frac{g}{2}\eta W^+_\mu|^2 +$$

$$+\frac{1}{2}|\partial_\mu\eta + iM_Z Z_\mu + \frac{ig}{2c_w}\eta Z_\mu|^2 - \sum_f \frac{g}{2}\frac{m_f}{M_W}\bar{\Psi}_f\Psi_f\eta$$

where $\Psi_f(x)$ is the Dirac spinor of fermion $f = e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau$, $\Psi_L(x) = (a_L^i(x), b_L^i(x)), i = 1, 2, 3$ denotes the $SU(2)_L$ doublet of left-handed fermions with a_L^i with $I_i^3 = 1/2$ (neutrinos) and b_L^i with $I_i^3 = -1/2$ (charged leptons), I_i^3 denotes the 3.component of the weak isospin, Q_f the electric charge, $s_w = \sin\theta_w$, $c_w = \cos\theta_w$ with θ_w denoting the weak mixing angle, g is the $SU(2)_L$ gauge coupling constant, g' is the $U(1)_Y$ gauge coupling constant, $e = \sqrt{4\pi\alpha}$ with α being the fine structure constant, $\eta(x)$ denotes the Higgs field, $A_\mu(x)$ the electromagnetic field, $W_\mu^\pm(x), Z_\mu(x)$ are the 3 weak gauge fields. m_f, M_W, M_Z, M_η denote the fermion, W boson, Z boson and Higgs boson masses, respectively,

The fermion masses in $\mathcal{L}_{GSW}$ result from their Yukawa interaction with the Higgs field ($\tilde{\Phi} = i\sigma^2\Phi^* = (\Phi^{0*}, -\Phi^-)$)

$$\mathcal{L}_{Yukawa} = \sum_{ij} c_{ij} \bar{\Psi}_L^i b_R^j \Phi + \sum_{ij} \tilde{c}_{ij} \bar{\Psi}_L^i a_R^j \tilde{\Phi} + h.c.$$

with $b_R^i, b^i = e, \tau, \mu$ are SU(2) singlets of right-handed leptons. With $\Phi = (0, v + \eta)/\sqrt{2}$ and after diagonalization of the mass matrix the fermion mass terms arise with $m_f = v/\sqrt{2}c_i^H, c_{ij} = \delta_{ij} c_i^H$.

The Higgs boson – a direct consequence of W and Z boson mass generation in the Standard Model via spontaneous symmetry breaking of the $SU(2)_L \otimes U(1)_Y$ gauge group. Goldstone (1961); Goldstone, Salam and Weinberg (1962); Higgs (1964,1966); Kibble (1967); Brout and Englert (1964); Guralnik, Hagen and Kibble (1964)

The Higgs particle so far eluded direct observation.

Where is the SM Higgs ?

We know from direct (LEP2) and indirect searches (EWK fits) that LHWG Note/2002-01, LEPEWWG April 2004

$$114.1\,\mathrm{GeV} < M_\eta \lesssim 251\,\mathrm{GeV}$$

Higgs discovery reach at the Tevatron Run II ($15\,\mathrm{fb}^{-1}$):

$$M_\eta \lesssim 180\ \mathrm{GeV}$$

Indirect searches via presence in loops: M_W - M_Z correlation:

$$M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)=\frac{\pi\alpha(0)}{\sqrt{2}G_\mu(1-\Delta r(M_W,m_t,M_\eta,\ldots))}$$

Global fit to all electroweak precision data: (LEPEWWG April 2004)

$$M_\eta < 251\,\mathrm{GeV}(95\%\mathrm{CL})\ ;\ M_\eta = 117^{+67}_{-45}\,\mathrm{GeV}$$