
4.2 The electroweak interaction of quarks

We still have to implement the hadronic sector into the Standard Model of electroweak interactions.

As discussed in the history section, we have to introduce for each left-handed lepton doublet a quark doublet to avoid anomalies that could spoil the renormalizability of the theory. Moreover we learned from experiment that the hadronic charged current is described by doublets consisting of a mixture of the down-type quarks (Cabibbo current). Thus we introduce 3 ($\times 3$ for each color degree of freedom, but here we suppress the color index) left-handed quark doublets

$$\Psi_L = \begin{pmatrix} a_L^i \\ b_L'^i \end{pmatrix} \quad a^i = u, c, t; b'^i = d', s', b'$$

with

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

V_{CKM} is a unitary matrix called the *Cabibbo-Kobayashi-Maskawa matrix* which is an extension of the Cabibbo matrix to a theory involving three generations of quarks. Moreover, there are six right-handed quarks ($\times 3$ for each color) which form a $SU(2)$ singlet $a_R^i, a^i = u, c, t; b_R^i, b^i = d, s, b$. This structure also guarantees that there are no strangeness-changing (in general no flavor-changing) neutral currents at the tree-level in the Standard Model.

Finally, we obtain the Lagrangian of the Standard Model describing the fundamental fermions, quarks and leptons, and their electroweak interaction by simply extending in \mathcal{L}_{GSW} (see lecture of 04/19) the sums \sum_f and \sum_i to also include the quarks: $i = 1, \dots, 6, b^i \rightarrow b'^i$, and $f = e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau, u, c, t, d, s, b$.

The CKM quark-mixing matrix can be parametrized in terms of three angles $\theta_i, i = 1, 2, 3$, and a phase δ as follows ($s_i = \sin \theta_i, c_i = \cos \theta_i$):

$$V_{CKM} = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ -s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix}$$

The complex phase introduces complex couplings of quarks to the W boson which can violate CP ($\mathcal{M}(CP(a + b \rightarrow c + d)) \neq \mathcal{M}^\dagger(a + b \rightarrow c + d)$). It is still an open question, if this is the (only) source for the observed CP violation in the neutral Kaon system.

CP violation - an asymmetry in matter and antimatter:

$K^0(d\bar{s}), \bar{K}^0(\bar{d}s)$ are produced in strong interaction processes (S is conserved), e.g.:

$$\pi^- p \rightarrow K^0 \Lambda^0 \quad \pi^+ p \rightarrow K^+ \bar{K}^0 p$$

They decay weakly (S is not always conserved), e.g.

$$K^0, \bar{K}^0 \rightarrow 2\pi, 3\pi, \pi^\pm l^\mp \nu_l$$

with $CP(2\pi) = +1, CP(3\pi) = -1(l = 0)$, i.e. K^0, \bar{K}^0 are not CP eigenstates ($CP|K^0 \rangle = -|\bar{K}^0 \rangle$ and $CP|\bar{K}^0 \rangle = -|K^0 \rangle$). K^0, \bar{K}^0 have neither definite mass nor decay width, i.e. they oscillate to each other and what decays are linear combinations of K^0 and \bar{K}^0

$$|K_1^0 \rangle = \frac{1}{\sqrt{2}}(|K^0 \rangle + |\bar{K}^0 \rangle) \quad \text{and} \quad |K_2^0 \rangle = \frac{1}{\sqrt{2}}(|K^0 \rangle - |\bar{K}^0 \rangle)$$

If CP invariance held, then K_1^0 would be CP even and K_2^0 CP odd and only $K_1 = K_S \rightarrow 2\pi$ (short lived) and $K_2 = K_L^0 \rightarrow 3\pi$ (long-lived) would occur, but Cronin and Fitch (1964 Brookhaven) found

$$K_L^0 \rightarrow \pi^+ \pi^-$$

Thus, the weak eigenstates are not CP eigenstates, but one can write the weak eigenstates as admixtures of CP eigenstates:

$$|K_S^0\rangle \propto (|K_1^0\rangle + \epsilon|K_2^0\rangle) \quad \text{and} \quad |K_L^0\rangle \propto (|K_2^0\rangle + \epsilon|K_1^0\rangle)$$

CP violation also occurs directly in the decays of the $K_{S,L}^0$ due to the decays of the $K_{1,2}^0$ components (direct CP violation). Thus, we distinguish between

- CP violation which occurs when two neutral mass eigenstates admixtures cannot be chosen to be CP eigenstates (indirect: $K_L \rightarrow \pi^+ \pi^-$ occurs since K_L^0 is not a state with $CP = -1$ (K_1^0 decays)), and
- CP violation in decay, which occurs in both charged and neutral decays, when the lifetime for a decay and its CP conjugate have different magnitude (direct: $K_L \rightarrow \pi^+ \pi^-$ occurs since CP is not conserved in the decay $K_L^0 \rightarrow \pi^+ \pi^-$ (K_2^0 decays)).

Both cases appear in the SM, but direct CP violation \ll indirect CP violation.

Direct CP violation in the decays of neutral Kaons is parametrized by the ϵ'/ϵ parameter

$$R = \frac{\frac{\Gamma(K_L \rightarrow \pi^+ \pi^-)}{\Gamma(K_S \rightarrow \pi^+ \pi^-)}}{\frac{\Gamma(K_L \rightarrow \pi^0 \pi^0)}{\Gamma(K_S \rightarrow \pi^0 \pi^0)}} = 1 + 6 \operatorname{Re}\left(\frac{\epsilon'}{\epsilon}\right)$$

Experiment (e.g. KTeV at Fermilab, NA48 at CERN): $\epsilon'/\epsilon = (1.8 \pm 0.4) \cdot 10^{-3}$.

Consistent with the SM expectation ?

Theory prediction obtained within the SM is plagued by large uncertainties: $0.14 \cdot 10^{-3} < \epsilon'/\epsilon < 3.2 \cdot 10^{-3}$.

CP violation effects due to complex CKM matrix elements should be more pronounced in the B^0, \bar{B}^0 meson system (see, e.g., the experiments BarBar at SLAC and Belle in Japan).