Isospin formalism

Consider NN scattering ($N = p, n$: isospin $I = 1/2$) and determine the relative probability for the scattering processes $pn \to d\pi^0$ and $pp \to d\pi^+$ (d=deuteron, a pn bound state with $I = 0, I_3 = 0$), i.e.

$$\sigma(pn \to d\pi^0) : \sigma(pp \to d\pi^+) = |A(pn \to d\pi^0)|^2 : |A(\sigma(pp \to d\pi^+)|^2 =?$$

Use that the strong interaction is invariant under $SU(2)$ transformations of the strong isospin (I):

- the transition amplitude (A_I) for a strongly interacting scattering process does depend on I but not on I_3,
- I and I_3 are conserved in these processes

$$< I'I'_3|T|II_3 >= \delta_{I_3 I'_3}\delta_{II'}A_I$$

Glebsch-Gordon coefficients:

$I(1) \otimes I(2)$	$I = 0$	$I = 1$		
$1/2 \otimes 1/2$	$I_3 = 0$	$I_3 = 1$	$I_3 = 0$	$I_3 = -1$
$I_3(1) = 1/2$	$\sqrt{1/2}$	1	$\sqrt{1/2}$	0
$I_3(1) = -1/2$	$-\sqrt{1/2}$	0	$\sqrt{1/2}$	1

Using the Glebsch-Gordon coefficients, the isospin states pn, pp are:

$$|pn >= |\frac{1}{2}\frac{1}{2}, \frac{1}{2} - \frac{1}{2} >= \sqrt{\frac{1}{2}}|10 > +\sqrt{\frac{1}{2}}|00 >$$

$$|pp >= |\frac{1}{2}\frac{1}{2}, \frac{1}{2}\frac{1}{2} >= |11 >$$

Moreover,

$$|\pi^+ d >= |11, 00 >= |11 > \quad |\pi^0 d >= |10, 00 >= |10 >$$

Thus, the transition amplitudes read as follows:

$$A_a =< \pi^0 d|T|pn >= \frac{1}{\sqrt{2}}(< 10|T|10 > + < 10|T|00 >) =< \pi^0 d|T|pn >= \frac{1}{\sqrt{2}} < 10|T|10 >$$

$$A_b =< \pi^+ d|T|pp >=< 11|T|11 >=< 10|T|10$$

so that

$$\sigma(pn \to d\pi^0) : \sigma(pp \to d\pi^+) = |\frac{1}{\sqrt{2}}|^2 = 1/2$$