4. The Glashow-Weinberg-Salam model of electroweak interactions

When constructing the Langrangian for the electroweak interaction among leptons and quarks, we have to keep in mind the phenomenology of the weak interaction, i.e. what we already have learned from experiment:

- At low energies, weak interaction processes are well described by a four-fermion point interactions, e.g. for β decay:

$$\mathcal{L} \sim \frac{G_F}{\sqrt{2}} J_\mu^{had}(x) J^{\mu,lept}(x)$$

- Parity violation $\Rightarrow V - A$ structure of the weak interaction:

$$J_\mu^{lept}(x) = \bar{\Psi}_e(x)\gamma_\mu(1-\gamma_5)\Psi_{\nu_e}(x)$$

- Universality: all weak currents couple with the same strength.

Because of the $V - A$ structure of the weak interaction, only left-handed electrons, $e_L = 1/2(1-\gamma_5)\Psi_e$, and neutrinos, $\nu_{e,L} = 1/2(1-\gamma_5)\Psi_{\nu_e}$, occur in β decay. With respect to the weak interaction e_L and $\nu_{e,L}$ are not distinguishable when neglecting the small mass difference. Thus, they can be viewed as manifestations of one weak state Ψ_L

$$\Psi_L(x) = \begin{pmatrix} \nu_{e,L} \\ e_L \end{pmatrix}$$

and the free theory, $\mathcal{L}_0 = \bar{\Psi}_L(x)(i \not{\partial})\Psi_L(x)$, is invariant under transformation of the global $SU(2)$ symmetry group of the weak isospin $I^a = \tau^a/2$:

$$\Psi_L(x) \rightarrow \Psi'_L(x) = U\Psi_L(x)$$

with $U = \exp[i/2\tau^a\alpha^a] = 1 + i/2\tau^a\delta\alpha^a, a = 1, 2, 3.$

But global symmetries are not considered "fundamental", since they contradict the idea of local gauge field theory.

In the construction of $\mathcal{L}$ of the weak interaction we identify the requirement to be invariant under a local $SU(2)$ transformation in the weak isospin space as the fundamental gauge principle.

We consider left-handed fermions to be grouped in the fundamental representation of the $SU(2)_L$ symmetry group of the weak isospin I_w. Right-handed electrons, $e_R = 1/2(1 + \gamma_5)\Psi_{\nu_e}$, form a $SU(2)_L$ singlet. We require the Lagrangian to be invariant under local $SU(2)_L$ transformations

$$\Psi_L(x) \to \Psi'_L(x) = U(x)\Psi_L(x)$$

$$e_R(x) \to e_R(x)$$

with

$$U(x) = \exp[\frac{i}{2}\tau^a\alpha^a(x)] = 1 + \frac{i}{2}\tau^a\delta\alpha^a(x)$$

The generators of the group $\tau^a, a = 1, 2, 3$ are the Pauli spin matrices which obey the following algebra

$$[\tau^a, \tau^b] = 2i\epsilon^{abc}\tau^c$$

where ϵ^{abc} are the $SU(2)_L$ structure constants.

As in the case of QED, gauge invariance requires the introduction of vector fields. We consider the gauge covariant derivative

$$\mathcal{D}_\mu = \partial_\mu + igW_\mu$$

with $W_\mu = 1/2\tau^a W^a_\mu, a = 1, 2, 3$ which introduces an interaction of the left-handed fermions with the $SU(2)_L$ vector fields W^a_μ. The kinetic term for the weak gauge fields reads

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu,a}$$

with the weak field strength tensor

$$F^a_{\mu\nu} = \partial\mu W^a_\nu - \partial\nu W^a_\mu - g\epsilon^{abc} W^b_\mu W^c_\nu$$

Together with a prescription how these gauge fields transform

$$W_\mu \to U(x) W_\mu U^\dagger(x) - \frac{i}{g} U \partial_\mu U^\dagger$$

we find that the Langrangian describing weak interactions among leptons

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I$$

with the free Lagrangian

$$\mathcal{L}_0 = \bar{\Psi}_L(x)(i\gamma^\mu \partial_\mu)\Psi_L(x) + \bar{e}_R(x) i\gamma^\mu \partial_\mu e_R(x)$$

$$+\frac{1}{2} W^a_\mu (g^{\mu\nu}\partial^2 - \partial^\mu \partial^\nu) W^a_\nu$$

and the interaction part

$$\mathcal{L}_I = \bar{\Psi}_L(x)(-g\gamma^\mu W_\mu)\Psi_L(x) + g\epsilon^{abc}(\partial^\mu W^{\nu,a})W_\mu^b W_\nu^c +$$

$$-\frac{1}{4}g^2(W^{\mu,a}W^{\nu,b}W_\mu^a W_\nu^b - W^{\mu,a}W^{\nu,a}W_\mu^b W_\nu^b)$$

is invariant under local $SU(2)_L$ transformations.

Let's have a closer look at the part of $\mathcal{L}_I$ that describes interactions of leptons with the gauge fields W_μ^a:

$$\mathcal{L}_{I,lep} = \bar{\Psi}_L(x)(-\frac{g}{2}\gamma^\mu\tau^a W_\mu^a)\Psi_L(x) =$$

$$= \bar{\Psi}_L(x)(-\frac{g}{2}\gamma^\mu)\begin{pmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -W_\mu^3 \end{pmatrix}\Psi_L(x) =$$

$$= -\frac{g}{2}\left\{\bar{\nu}_{e,L}\gamma^\mu W_\mu^3 \nu_{e,L} + \sqrt{2}\bar{\nu}_{e,L}\gamma^\mu W_\mu^+ e_L + \right.$$

$$\left. + \sqrt{2}\bar{e}_L\gamma^\mu W_\mu^- \nu_{e_L} - \bar{e}_L\gamma^\mu W_\mu^3 e_L\right\}$$

with

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp i W_\mu^2)$$

The vector field W_μ^3 cannot be identified with the photon, since it also couples to left-handed neutrinos.

Thus, as a consequence of the gauge principle, a new form of weak interaction arises:

The GWS model predicts the existence of a neutral weakly interacting boson, that mediates a weak interaction (neutral currents).

However, we still need to include the electromagnetic interaction and the fact that electrons and the weak gauge bosons are massive.