The fundamental constituents of matter are fermions (leptons and quarks), i.e. particles with spin $\frac{1}{2}$. A scalar field cannot describe a particle with spin. To describe non-relativistic electrons Pauli suggested to use a wavefunction with two components

$$\Psi(x) = \begin{pmatrix} \Psi_1(x) \\ \Psi_2(x) \end{pmatrix}$$

where $|\Psi_{1,2}(x)|^2 d^3x$ is the probability to find an electron with spin in positive(negative) z-direction in the volume d^3x around the point $\vec{x}$. The angular momentum operator is given by $\vec{J} = \vec{L} + \vec{S}$ ($\vec{L} = \vec{x} \times \vec{p}, \vec{p} = 1/i\vec{\partial}$) with the spin angular momentum $\vec{S} = 1/2\vec{\sigma}$ where $\sigma_i, i = 1, 2, 3$ are the three Pauli spin matrices. According to Pauli the wave function for a free electron should obey the Schroedinger equation. However, the Schroedinger equation is not relativistic invariant.

Dirac's ansatz to describe a relativistic free electron is ($\partial_\mu = \partial/\partial x^\mu = (\partial/\partial t, \vec{\partial})$)

$$(i\gamma^\mu \partial_\mu - a)\Psi(x) = 0$$

where the nature of the coefficient γ_μ and the constant a are not specified yet.

Energy and momentum of the electron fulfill the relativistic energy-momentum relation

$$E^2 - \vec{p}^2 = m^2$$

where m is the mass of the electron. Thus, the components of the field function Ψ, Ψ_i, should obey the Klein-Gordon equation.

Let's linearize the Klein-Gordon equation as follows:

$$(-i\gamma^\mu \partial_\mu - m)(i\gamma^\nu \partial_\nu - m)\Psi_i(x) = 0$$

so that

$$(1/2(\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu)\partial_\mu\partial_\nu + m^2)\Psi_i(x) = 0$$

This equation is identical with the Klein-Gordon equation if

$$\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$$

Dirac found that this relation can be fulfilled when asumming that γ_μ are 4×4 matrices. In the Dirac representation these Dirac matrices read (1 denotes the 2×2 unit matrix):

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \gamma^j = \begin{pmatrix} 0 & \sigma^j \\ -\sigma^j & 0 \end{pmatrix}$$

Finally, Dirac postulates the following equation for a relativistic electron

$$(i\gamma^\mu \partial_\mu - m)\Psi(x) = 0$$

where $\Psi(x)$ is a field function with four components $\Psi_i(x)$ which each fullfil the Klein-Gordon equation.

$\Psi(x)$ is called a Dirac spinor, i.e. it transforms under transformations of the Lorentz (Poincare) group as follows

$$x^\mu \rightarrow x^{'\mu} = \Lambda^\mu_\nu x^\nu + a^\mu$$

$$\Psi(x) \rightarrow \Psi'(x') = S(\Lambda)\Psi(x)$$

The Dirac matrices transform as four-vectors

$$\gamma^{'\mu} = \Lambda^\mu_\nu \gamma^\nu = S^{-1}\gamma^\mu S$$

One can show that there exists such a matrix $S(\Lambda)$ so that $\Psi'(x')$ also fulfills the Dirac equation

$$(i\gamma^\mu \partial'_\mu - m)\Psi'(x') = 0$$