Homework IV

Consider the production of a muon-anti-muon pair in the high-energy collision of an electron and positron at center-of-mass energies below the Z^0 boson mass, i.e. where it is a good approximation to assume that only photon exchange contributes. Then we can calculate the tree-level cross section by only using QED Feynman rules.

Follow the steps outlined below to calculate the scattering cross section to the process $e^-(p_4, s_4)e^+(p_3, s_3) \to \mu^-(p_2, s_2)\mu^+(p_1, s_1)$ at lowest order in perturbation theory (p_i, s_i denote the four-momenta and spins of the particles) assuming that the polarization of ingoing and outgoing particles is not measured (unpolarized cross section). If the latter is the case, one has to sum(take the average) over all final(initial) state spin degrees of freedom. Use the Feynman rules as provided in the lecture notes. You can either set the fermion masses to zero or try the calculation with non-zero fermion masses.

i.) Draw all Feynman diagrams contributing to this process at lowest order perturbation theory.

ii.) Write down the corresponding analytic expression of the scattering matrix element $\mathcal{M}$.

iii.) Calculate $\overline{\sum}|\mathcal{M}|^2$ and the differential cross section $d\sigma/d\Omega$. You will need the following spin sums for fermions:

$$\sum_s u(k,s)\bar{u}(k,s) = \gamma^\mu k_\mu + m$$

$$\sum_s v(k,s)\bar{v}(k,s) = \gamma^\mu k_\mu - m$$

Due on Wednesday, April 12, 2006 (in class).

The traces of 2 and 4 gamma matrices in 4 space-time dimensions read

$$Tr[\gamma_\mu \gamma_\nu] = 4g_{\mu\nu}$$

$$Tr[\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma] = 4g_{\mu\sigma}g_{\nu\rho} - 4g_{\mu\rho}g_{\nu\sigma} + 4g_{\mu\nu}g_{\rho\sigma}$$

Traces of odd numbers of gamma matrices are zero.