Homework III

1. Show that the manifestly covariant form of the inhomogeneous Maxwell equations

$$\partial_\mu F^{\mu\nu} = j^\nu$$

is equivalent to

$$\vec{\nabla} \cdot \vec{E} = \rho \quad \text{and} \quad \vec{\nabla} \times \vec{B} = \vec{j} + \partial_t \vec{E}$$

2. Consider the Lagrange function for a massive, charged particle that moves with velocity $\vec{v}$ in an electromagnetic field. Show that the Lagrange equation of motion

$$L = \frac{1}{2} m v^2 - q\Phi + q\vec{A} \cdot \vec{v}$$

is equivalent to

$$m\vec{a} = q[\vec{E} + \vec{v} \times \vec{B}]$$

It is sufficient to show this for only the x component, for instance. Note: $\vec{B} = \vec{\nabla} \times \vec{A}, \vec{E} = -\vec{\nabla}\Phi - \partial_t \vec{A}$.

3. Describe in your own words Emmy Noether's theorem and how it is used to incorporate (electric) charge conservation in the theory of a free charged, spinless particle, whose motion in space-time is described by the Klein-Gordon equation for a complex, scalar field (see lecture notes).

Now consider a local phase transformation, i.e. $\alpha \to \alpha(x)$. Does this transformation still leave the Lagrangian for the free Klein-Gordon field form invariant ?

What do you suggest to do to make the Langragian form invariant even under local phase transformation of the fields ?

Due on Friday, March 31st, 2006 (in class).