Homework I

Consider pion-nucleon (πN) scattering (π: isospin $I = 1$, $N = p, n$: isospin $I = 1/2$) and determine the relative probability for the scattering processes $p\pi^+ \to p\pi^+$, $p\pi^- \to p\pi^-$, and $p\pi^- \to n\pi^0$, i.e.

$$\sigma(p\pi^+ \to p\pi^+) : \sigma(p\pi^- \to p\pi^-) : \sigma(p\pi^- \to n\pi^0) =$$
$$= |A(p\pi^+ \to p\pi^+)|^2 : |A(p\pi^- \to p\pi^-)|^2 : |A(\sigma(p\pi^- \to n\pi^0)|^2 = ?$$

Use that the strong interaction is invariant under $SU(2)$ transformations of the strong isospin (I):

- the transition amplitude (A_I) for a strongly interacting scattering process does depend on I but not on I_3,
- I and I_3 are conserved in these processes:

$$A_I = \delta_{I_3 I'_3} \delta_{II'} < I' I'_3 | T | I I_3 >$$

Note: $|A_{3/2}| >> |A_{1/2}|$, i.e. neglect $I = 1/2$ resonances.

Glebsch-Gordon coefficients:

$I(1) \otimes I(2)$	$I = 1/2$		$I = 3/2$			
$1 \otimes 1/2$	$I_3 = 1/2$	$I_3 = -1/2$	$I_3 = 3/2$	$I_3 = 1/2$	$I_3 = -1/2$	$I_3 = -3/2$
$I_3(2) = 1/2$	$-\sqrt{1/3}$	$-\sqrt{2/3}$	1	$\sqrt{2/3}$	$\sqrt{1/3}$	0
$I_3(2) = -1/2$	$\sqrt{2/3}$	$\sqrt{1/3}$	0	$\sqrt{1/3}$	$\sqrt{2/3}$	1