

---

## Electric Circuits

An electric circuit consists of a source for the electromotive force, e.g., a battery, and devices (resistors, capacitors, etc.) that are connected to the battery via conducting wires. We usually neglect the resistance of the wires compared to the much larger resistance of the devices. Due to the potential difference at the battery (=voltage  $V$ ) a current,  $I$ , flows through the circuit and delivers electric power to the devices in the circuit:  $P = IV$  (in [W]). The voltage needed to push a certain current through a device is given by **Ohm's law** (assume  $R = \text{constant}$ ):  $V = RI$ .

The **power consumed by a resistor** is  $P = I^2R = V^2/R$ .

### Resistors wired in series:

When resistors with resistances  $R_1, R_2$ , etc. are wired in series in a circuit the current through each resistor is the same. The battery voltage,  $V$ , however, is divided up among the resistors as follows

$$V = V_1 + V_2 + \dots = R_1I + R_2I + \dots = I(R_1 + R_2 + \dots) \Rightarrow V = IR_s$$

with the **equivalent resistor** in series wiring,  $R_s = R_1 + R_2 + \dots$

---

### Resistors wired in parallel:

When resistors with resistances  $R_1, R_2$ , etc. are wired in parallel the voltage across each resistor is the same. The battery current,  $I$ , however, is divided up among the resistors as follows

$$I = I_1 + I_2 + I_3 \dots = \frac{V}{R_1} + \frac{V}{R_2} + \dots = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \dots\right) \Rightarrow I = \frac{V}{R_p}$$

with the **equivalent resistor** in parallel wiring,  $1/R_p = 1/R_1 + 1/R_2 + \dots$

### Capacitors wired in series:

The charge  $q$  on each capacitor is the same. The battery voltage, however, is divided up among the capacitors as follows

$$V = V_1 + V_2 + V_3 \dots = \frac{q}{C_1} + \frac{q}{C_2} + \dots = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \dots\right) = \frac{q}{C_s}$$

with the equivalent capacitor in series wiring,  $1/C_s = 1/C_1 + 1/C_2 + \dots$

---

### Capacitors wired in parallel:

The voltage across each capacitor is the same. The total charge stored by all capacitors, however, is divided up among the capacitors as follows

$$q = q_1 + q_2 + \dots = VC_1 + VC_2 + \dots = V(C_1 + C_2 + \dots) = VC_p$$

with the equivalent capacitor in parallel wiring,  $C_p = C_1 + C_2 + \dots$

An electric circuit may be considerably simplified by reducing it to equivalent resistors and capacitors. For complicated electric circuits one applies Kirchhoff's rules:

1. The sum of the currents entering any junction must equal the sum of the currents leaving that junction (**junction rule**)
2. The sum of the voltages across all elements around any closed circuit loop must be zero (**loop rule**).

---

When applying Kirchhoff's rules you must make two decisions at the beginning of the problem:

1. You must assign symbols and directions to the currents in all branches of the circuit.
2. When applying the loop rule, you must choose a direction for going around the loop. Write down voltage drops and rises according to the following rules:
  - a) If a resistor is traversed in the direction of the current, the change in voltage across the resistor is  $-IR$  (drop).
  - b) If a resistor is traversed in the direction opposite the current, the change in voltage across the resistor is  $+IR$  (rise).
  - c) If a source of emf is traversed in the direction of the emf, (from  $-$  to  $+$  on the terminals), the change in voltage is  $+V$  (rise).
  - d) If a source of emf is traversed opposite the direction of the emf, (from  $+$  to  $-$  on the terminals), the change in voltage is  $-V$  (drop).

---

## Magnetic Forces and Magnetic Fields

**Magnets** are mostly metallic objects which attract other metal-containing objects. There are two magnetic poles: north pole and south pole. Magnetic poles exert attractive or repulsive forces on each other: Like poles repel each other and unlike poles attract each other. To describe the interactions of magnets and magnetized materials, it is convenient to introduce the concept of the magnetic field: The **magnetic force exerted on a charge**  $q_0$ , **moving with velocity**  $\vec{v}$  can be used to describe the properties of the magnetic field,  $\vec{B}$ :

$$B = \frac{F}{q_0 v \sin \theta} \quad [T]$$

**Direction of the magnetic force:** The direction of the magnetic force is always perpendicular to both  $\vec{v}$  and  $\vec{B}$ . Use of the right hand rule: Hold your right hand open with your fingers pointing in the direction of  $\vec{B}$  and your thumb pointing in the direction of  $\vec{v}$ .  $\vec{F}$ , on a positive charge, is then directed **out** of the palm of your hand. For a negative charge reverse the direction of  $\vec{F}$ .

---

## Motion of a charged particle in a magnetic field

Consider a positively charged particle with mass  $m$  and charge  $q_0$  moving in a uniform magnetic field so that the direction of the particle's velocity,  $\vec{v}$  is perpendicular to the magnetic field. The magnetic force (right-hand rule!) is always directed toward the center of a circular path  $\rightarrow$  the magnetic force,  $\vec{F}$ , is effectively a centripetal force,  $\vec{F}_c$ :

$$\vec{F} = \vec{F}_c \quad \text{with} \quad F = q_0 v B \quad \text{and} \quad F_c = \frac{mv^2}{r}$$

which gives for the radius  $r$  of the circular path

$$r = \frac{mv}{q_0 B}$$

The time it takes to travel once around the circular path is given by the period

$$T = \frac{2\pi r}{v}$$

**(Application: Mass spectrometer)**

---

### Magnetic force on a current-carrying conductor

A current - carrying wire with length  $\ell$  in an external magnetic field feels a magnetic force:

$$F = BI\ell \sin \theta$$

The direction of the force can be determined using the right-hand rule with the thumb pointing in the direction of the current.

### Magnetic field of a long straight conductor

A current - carrying wire produces a magnetic field. The magnetic field lines around a wire form concentric circles. The magnitude of the field in the distance  $r$  of the wire is

$$B = \frac{\mu_0 I}{2\pi r}$$

where  $\mu_0$ , is called the **permeability of free space**. The direction of the magnetic field can be determined by the right hand rule:

If the wire is grasped in the right hand with the thumb in the direction of the current, the fingers will curl in the direction of  $\vec{B}$ .

---

### Magnetic force between two parallel wires

Since a current in a conductor creates its own magnetic field, two current-carrying wires placed close together a distance  $d$  apart exert magnetic forces on each other: The magnetic force on wire 1 (length:  $\ell$ ) due to the magnetic force created by wire 2,  $B_2$ , is:

$$F_1 = B_2 I_1 \ell = \left( \frac{\mu_0 I_2}{2\pi d} \right) I_1 \ell = \frac{\mu_0 I_1 I_2 \ell}{2\pi d}$$

If  $I_1$  and  $I_2$  flow in the same direction, the two wires attract each other; if the currents flow in opposite directions, the force between the wires is repulsive.

### Magnetic field of a current loop and a solenoid

A **solenoid** is a coil of several closely spaced loops. They act as magnets when they carry a current,  $I$ . The magnetic field inside a solenoid is **uniform**, and its magnitude is given by

$$B = \mu_0 n I$$

where  $n = N/\ell$  is the number of turns per unit length.

---

## Electromagnetic inductance

A changing magnetic field produces an electric current. In order to understand this phenomenon, we need the concept of **magnetic flux**: Consider a loop of wire in the presence of a uniform magnetic field,  $\vec{B}$ . The loop has an area  $A$ . The magnetic flux,  $\Phi$ , is defined as

$$\Phi = BA \cos \phi \quad [Wb]$$

where  $\phi$  is the angle between  $\vec{B}$  and the normal (perpendicular) to the loop. If the field lines are parallel to the loop:  $\phi = 90^\circ$  and  $\Phi = 0$ . When the field lines are perpendicular to the loop:  $\phi = 0$  and  $\Phi = BA$ .

## Faraday's law of induction:

The average emf (=voltage),  $\varepsilon$ , induced in a circuit containing  $N$  loops equals the rate of change of magnetic flux through the circuit:

$$\varepsilon = -N \frac{\Delta\Phi}{\Delta t} = -N \frac{\Phi_f - \Phi_i}{t_f - t_i} \quad [V]$$

*The induced emf produces a current whose magnetic field opposes the change in magnetic flux through the loop. (Lenz's law)*

---

## Motional emf

Consider a straight conductor of length  $\ell$  moving with constant velocity into an area of a uniform magnetic field perpendicular to the magnetic field lines. If the moving conductor is part of a closed conducting path a (motional) emf is created and a current is flowing. Assume that the conductor is moving a distance  $\Delta x$  in a time interval  $\Delta t$ . The change of flux is then:  $\Delta\Phi = B\Delta A = B\ell\Delta x$ . One then finds for the **motional emf** (magnitude):

$$\varepsilon = \frac{\Delta\Phi}{\Delta t} = B\ell \frac{\Delta x}{\Delta t} = B\ell v$$

Direction of the induced current: Lenz's law  $\Rightarrow$  the magnetic force exerted on the induced current by the external magnetic field has to oppose the direction of motion of the loop.

## Self-inductance

The phenomenon that a changing current in a circuit induces an emf in the same circuit, is called **self-inductance**.

---

The self-induced emf is proportional to the time rate of change of the current:  $\varepsilon = -L \frac{\Delta I}{\Delta t}$ , where  $L$  is called the **inductance** of the device:

$$L = \frac{N\Phi}{I} \quad [H]$$

Lenz's law  $\Rightarrow$  the induced emf in the circuit is opposite to the direction of the current.

### Mutual inductance

The effect that a changing current in one circuit (=primary circuit) induces an emf in another circuit (=secondary circuit) is called **mutual inductance**. The magnetic flux in the secondary circuit,  $\Phi_s$ , is proportional to the alternating magnetic field, which is proportional to the alternating current in the primary circuit:  $N_s \Phi_s = M I_p$ .  $M$  is called the **mutual inductance** of the device. Thus, the emf due to mutual inductance is proportional to the time rate of change of the primary current:  $\varepsilon_s = -M \frac{\Delta I_p}{\Delta t}$ .

**(Application: Transformers:  $V_s/V_p = N_s/N_p$ )**

---

### Resistors in an AC Circuit

An AC circuit consists of combinations of circuit elements and an AC generator which provides an AC current (i.e. the emf changes sinusoidally with time). The instantaneous voltage is given by  $V = V_0 \sin(2\pi ft)$ , where  $V_0$  is the maximum voltage, and  $f$  is the frequency at which the voltage changes (measured in Hertz;  $1 \text{ Hz} = 1 \text{ s}^{-1}$ ).

**Ohm's law also holds in an AC circuit:**  $V = IR$ . Thus, the instantaneous current through a resistor is given by  $I = I_0 \sin(2\pi ft)$ , where  $I_0 = V_0/R$  is the maximum current. The average (*rms*) voltage and current is given by their peak values as

$$I_{rms} = \frac{I_0}{\sqrt{2}} \quad \text{and} \quad V_{rms} = \frac{V_0}{\sqrt{2}}$$

### Inductors in an AC Circuit

An inductor is a device that operates on the basis of Faraday's law of electromagnetic induction, The effective resistance of an inductor in an AC circuit is measured by the **inductive reactance**,  $X_L$ , defined by

$$X_L = 2\pi fL \quad [\Omega]$$

---

The analog of Ohm's law for the average voltage across an inductor is:

$$V_{rms} = I_{rms} X_L.$$

### Capacitors in an AC Circuit

Consider a circuit consisting of a capacitor and an AC generator. The impeding effect of a capacitor on the current in an AC circuit is expressed in terms of a factor called the **capacitive reactance**,  $X_C$ , defined as

$$X_C = \frac{1}{2\pi f C} \quad [\Omega]$$

Analog form of Ohm's law for a capacitor in an AC (!) circuit:  $V_{rms} = I_{rms} X_C$ . For  $f = 0$  (DC circuit),  $X_C = \infty$  and the current is zero.

### The RLC Series Circuit

A series RLC circuit contains a resistor, an inductor, and a capacitor connected in series across an AC generator: The net instantaneous voltage,  $V$ , across all three elements is the sum of the three individual instantaneous voltages:  $V = V_R + V_C + V_L$ . The average voltage is given by

$$V_{rms} = \sqrt{V_R^2 + (V_L - V_C)^2}$$

---

The phase angle is given by:  $\tan \Phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}$ . Defining the **impedance**,  $Z$ , by

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad [\Omega]$$

we find the analog of **Ohm's law for an RLC circuit**:  $V_{rms} = I_{rms} Z$ .

### Power in an AC Circuit

The only element in an RLC circuit which dissipates energy is the resistor. The average power lost in a resistor is  $\bar{P} = I_{rms}^2 R = I_{rms} V_R$ , where  $V_R$  can be expressed in terms of the voltage of the source and the phase angle:  $V_R = V_{rms} \cos \Phi$ .

### Resonance in a series RLC circuit

The current has its maximum when  $Z$  is at a minimum. This happens for  $X_C = X_L$ , and thus  $Z = R$ . The frequency  $f = f_0$  at which it happens is called the **resonance frequency** of the circuit:

$$2\pi f_0 L = \frac{1}{2\pi f_0 C} \Rightarrow f_0 = \frac{1}{2\pi \sqrt{LC}}$$

---

## Electromagnetic Waves

Maxwell predicted that *light waves are electromagnetic in nature*. In other words, light consists of fluctuating electric and magnetic fields that travel through space as transverse waves:

- The electric and magnetic fields in an electromagnetic wave are in phase and are perpendicular to each other.
- The electric and magnetic fields oscillate in planes perpendicular to the direction of travel of the wave (transverse waves).
- Electromagnetic waves travel in vacuum at the speed of light:

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 2.99792 \times 10^8 \text{ m/s.}$$

- The ratio of the electric to the magnetic field in an electromagnetic field is given by  $\frac{E}{B} = c$ .
- The frequency,  $f$ , and wavelength  $\lambda$  of electromagnetic waves is related by:  $c = f\lambda$ .

- 
- Electromagnetic waves carry energy. The total energy density,  $u$ , carried by an electromagnetic wave is

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} B^2$$

For the average energy density,  $\bar{u}$ , replace  $E, B$  by  $E_{rms}, B_{rms}$ .

To describe the energy transport by an electromagnetic wave we introduce *intensity*, which is the power passing through an unit area perpendicular to the travel direction per unit area: ( $S = P/A$ ):

$$S = cu \quad \left[ \frac{W}{m^2} \right]$$

Note: Average intensity  $\bar{S} = c\bar{u}$ .

## Polarization of Electromagnetic Waves

Electromagnetic waves emitted by an ordinary source are **unpolarized**, *ie* the electric field vector,  $\mathbf{E}$ , points randomly in the plane perpendicular to the direction of wave travel.

---

A wave is said to be **linearly polarized** if  $\mathbf{E}$  oscillates in the same direction at all times.

- Suppose an unpolarized light beam is incident on a **polarizer**. The light that is passing through is polarized linearly in the direction of the polarization axis of the polarizer. **Its intensity is reduced to half its original value.**
- A 2nd polarizer (called **analyzer**) intercepts this beam with its transmission axis at an angle  $\theta$  with respect to the axis of the first polarizer.
- The component of  $\mathbf{E}$  perpendicular to the axis of the analyzer is completely absorbed, while the component parallel to the axis passes through. The average transmitted intensity is

$$\bar{S} = \bar{S}_0 \cos^2 \theta$$

where  $\bar{S}_0$  is the average intensity of the polarized wave incident on the analyzer (**Malus' law**).

---

### The Doppler effect

When electromagnetic waves, the source and the observer all travel in vacuum (or air) the observed frequency,  $f_o$ , compared to the frequency emitted by the source,  $f_s$ , of the electromagnetic wave will depend on the relative speed of the source and the observer (for  $v_{rel} \ll c$ ):

$$f_o = f_s \left( 1 \pm \frac{v_{rel}}{c} \right)$$

+: source and observer come together

–: source and observer move apart