

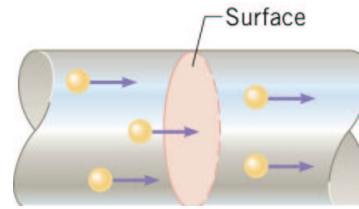
Electric Circuits

1 – Electric Current and Electromotive Force

The flow of electric charges: Electric currents power light bulbs, TV sets, computers etc.

Definition of **electric current**:

The current is the *rate at which charge flows through a surface perpendicular to the flow*:



$$I = \frac{\Delta q}{\Delta t} \quad (1)$$

SI unit: **ampere (A)**: $1 A = 1 C/s$

- It is conventional to give the current the same direction as the flow of positive charge.

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Circuits:

The transfer of electric energy in form of kinetic energy of the moving charge carriers takes place via electric circuits. A circuit consists of an energy source (e.g., battery, generator) and energy consuming devices (e.g., light bulb, TV, computer) which are connected by conducting wires.

“**emf**”: source that maintains a constant current in a closed circuit. The name originates from *electromotive force*. The **emf** is the maximum potential difference provided by the source.

direct current (dc) circuits: charges move around circuits in the same direction at all times (battery).

alternating current (ac) circuits: the direction of charges moving around a circuit changes from moment to moment (generators at power companies).

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2 – Resistance and Ohm's Law

When a voltage, V , is applied across a piece of material, the current, I , is found to be proportional to the applied voltage. The ratio V/I is called the resistance, R , of the material.

- For many materials, including most metals, the resistance is constant over a wide range of applied voltages. When the resistance is constant Ohm's law applies:

$$\frac{V}{I} = R = \text{constant} \quad \text{or} \quad V = RI \quad (2)$$

SI unit: **ohm** (Ω): $1 \Omega = 1 \text{ V/A}$

- Ohm's law is an empirical relationship.

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3 – Resistivity

In a metal conductor, the electric current is carried by moving electrons. Resistance originates from collisions of these electrons with atoms.

The resistance of an ohmic conductor is proportional to its length, L , and inversely proportional to its cross-sectional area A :

$$R = \rho \frac{L}{A} \quad (3)$$

- ρ is called the **resistivity** of the material.
- ρ depends on the electronic structure of the material and the temperature: good conductors have a small resistivity, good insulators have a high resistivity.
- Eq. 3 is analogous to the flow of a liquid through a pipe.

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Temperature variation of resistance:

For most metals, resistivity increases approximately linearly with temperature over a limited temperature range:

$$\rho = \rho_0 [1 + \alpha(T - T_0)] \quad (4)$$

ρ_0 is the resistivity at a reference temperature T_0 (usually 20°C), ρ is the resistivity at a temperature T , and α is the **temperature coefficient of resistivity**.

Since R is proportional to ρ :

$$R = R_0 [1 + \alpha(T - T_0)] \quad (5)$$

$\alpha > 0$: R increases with increasing temperature (metals)

$\alpha < 0$: R decreases with increasing temperature (semiconductors)

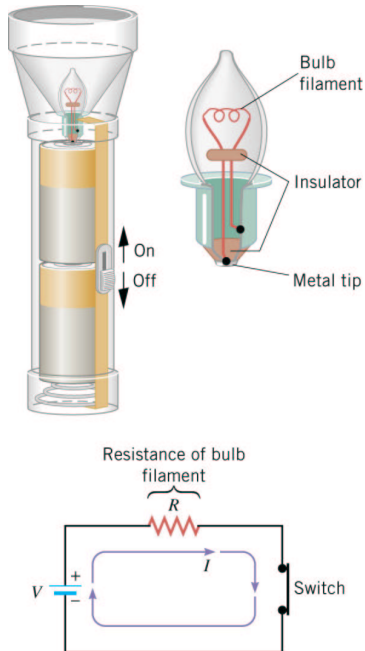
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4 – Superconductors

- There is a class of metals and compounds whose resistances suddenly go virtually to **zero** below a critical temperature T_c .
- These materials are called **superconductors**.
- Once a current is set up in a superconductors, it persists *without any applied voltage* ($\rightarrow R = 0$)
- For most metals, T_c is typically a few $^\circ\text{K}$.
- For certain compounds such as $YBa_2Cu_3O_7$, T_c is about 100°K . These materials are called **high T_c superconductors**.
- Is there superconductivity at room temperature??

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5 – Electrical Energy and Power



Battery: chemical energy stored in the battery is continuously transformed into kinetic energy of the charge carriers through the electric potential difference at the + and – terminals of the battery. This kinetic energy is quickly lost due to collisions between the charge carriers and the atoms (ions) in a resistor.
→ temperature of conductor increases
→ chemical energy is transformed into thermal energy

(Note: we neglect the resistance of the thin wires connecting the battery and the resistor)

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Rate at which a charge Δq loses energy when it passes through a resistor (V is the voltage):

$$\frac{\Delta q}{\Delta t} V = IV \quad (6)$$

Rate of energy change = Power (P) \Rightarrow

$$P = IV \quad (7)$$

or with Ohm's law ($V = RI$)

$$P = I^2 R = \frac{V^2}{R} \quad (8)$$

Example: space heater. $P = 2 \text{ kW}$, $V = 120 \text{ V}$, operating for 5 min.

$$\rightarrow R = \frac{V^2}{P} = 7.2 \Omega$$

$$\rightarrow I = \sqrt{\frac{P}{R}} = 16.7 \text{ A}$$

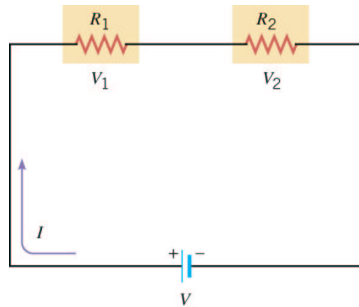
$$\rightarrow \text{energy} = P\Delta t = 2 \text{ kW} \cdot 300 \text{ s} = 6 \cdot 10^5 \text{ J} = 0.167 \text{ kWh}$$

$$\text{Conversion factor: } 1 \text{ kWh} = 3.6 \times 10^6 \text{ J}$$

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6 – Resistors in Series

Consider two resistors, R_1 and R_2 connected to a battery in a circuit called a series circuit:



series circuit: there is only one pathway for the electric current.

Total potential difference:

$$V = IR_1 + IR_2 \quad (9)$$

Note: the same current, I , is flowing through both resistors (because we consider a series circuit).

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R_1 and R_2 can be replaced with an equivalent resistor, R_s . Apply Ohm's law:

$$V = IR_s = IR_1 + IR_2 \quad (10)$$

→

$$R_s = R_1 + R_2 \quad (11)$$

For three or more resistors connected in series:

$$R_s = R_1 + R_2 + R_3 + \dots \quad (12)$$

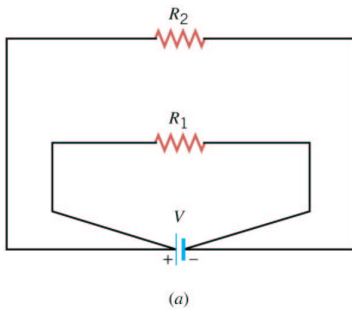
Note:

The equivalent resistance of a series combination of resistors is always greater than any individual resistance.

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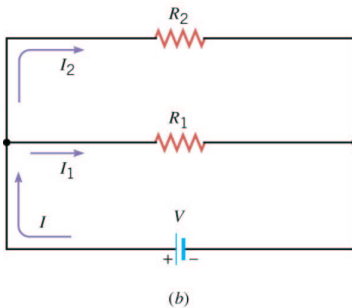
7 – Resistors in Parallel

Consider two resistors, R_1 and R_2 connected in parallel:



(a)

When resistors are connected in parallel, the potential differences (voltages) across them are the same.



(b)

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Since charge is conserved, the current, I , which enters point **a** must equal the total current leaving that point, $I_1 + I_2$:

$$I = I_1 + I_2 \quad (13)$$

Apply Ohm's law to each resistor: $I_1 = \frac{V}{R_1}$, $I_2 = \frac{V}{R_2}$
and to the equivalent resistor:

$$I = \frac{V}{R_p} = \frac{V}{R_1} + \frac{V}{R_2} \quad (14)$$

Therefore:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} \quad (15)$$

For 3 or more resistors in parallel:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots \quad (16)$$

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Remarks:

- The equivalent resistance of 2 or more resistors connected in parallel is always *less* than the smallest resistance in the group.
- Household circuits are always wired so that appliances are connected in parallel.
→ Each device operates independently, and each receives the same voltage.

Problem-solving Strategy

Resistors

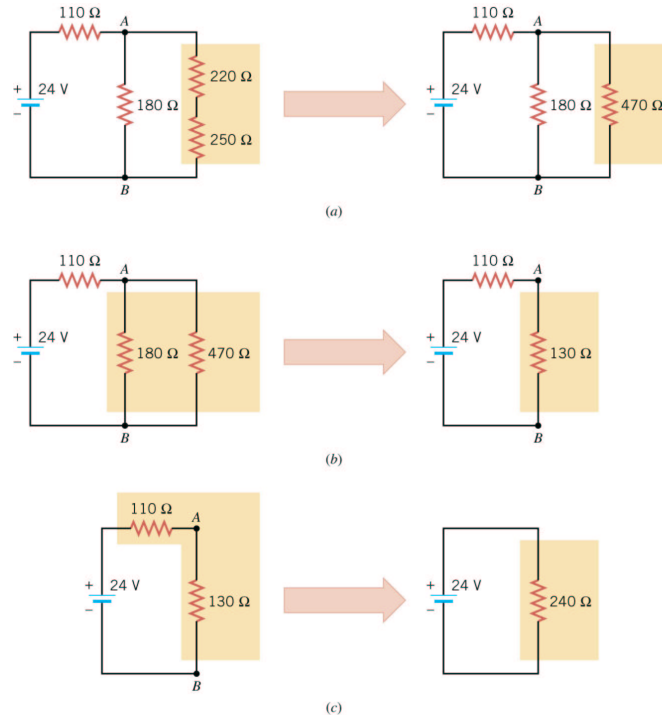
1. When 2 or more resistors are connected in **series**, they carry the same current, but the voltages across them are different. The resistances add directly to give the equivalent resistance of the series combination.
2. When 2 or more resistors are connected in **parallel**, the voltages across them are the same. The equivalent resistance of a parallel combination of resistors is found through **reciprocal** addition.

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3. A complicated circuit consisting of resistors can often be reduced to a simple circuit with only one equivalent resistor. To do so:
 - a) replace any resistors in series or any in parallel using step 1. or 2.
 - b) Sketch the new circuit.
 - c) repeat a) if necessary.
 - d) Continue this process until a single equivalent resistance is found.
4. If the current through or the voltage across a resistor in the complicated circuit is to be identified, start with the final circuit of 3. and gradually work back, using $V = IR$ and the rules of 1. and 2.

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Example:



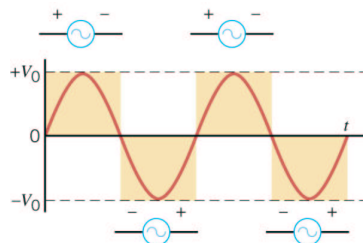
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8 – Resistors in an AC Circuit

An AC circuit consists of combinations of circuit elements and an AC generator which provides an AC current (i.e. the emf changes sinusoidally with time). The instantaneous voltage is given by

$$V = V_0 \sin(2\pi ft)$$

where V_0 is the maximum voltage, and f is the frequency at which the voltage changes (measured in Hertz; $1 \text{ Hz} = 1 \text{ s}^{-1}$).



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Now consider a circuit consisting of a resistor and an AC generator:

- Ohm's law holds in an AC circuit: $V = IR$. Thus, the instantaneous current is given by

$$I = I_0 \sin(2\pi ft)$$

where $I_0 = V_0/R$ is the maximum current.

- The direction of the current has no effect on the behavior of the resistor. The rate at which energy is converted into heat in a resistor (*ie* the power) is proportional to the square of the instantaneous current, I :

$$P = I^2 R = I_0 V_0 \sin^2(2\pi ft)$$

- The average power lost in a resistor in an AC circuit is given by

$$\bar{P} = \frac{1}{2} I_0^2 R = I_{rms}^2 R$$

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where

$$I_{rms} = \frac{I_0}{\sqrt{2}} \quad (17)$$

is the **rms current** (*rms* stands for *root mean square*). **Meaning:** An alternating current with a maximum current I_0 produces the same power loss in a resistor as a direct current $I_{rms} = I_0/\sqrt{2}$.

- In a similar fashion one defines the **rms voltage**

$$V_{rms} = \frac{V_0}{\sqrt{2}} \quad (18)$$

- The average power lost in a resistor in an AC circuit is then also given by

$$\bar{P} = I_{rms} V_{rms} \quad (19)$$