1-loop Calculations for $t\bar{t}$ Pair Production at Hadron Colliders

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1-loop Calculations for $t\bar{t}$ Pair Production at Hadron Colliders

Outline

- Properties of Top Quark
- Polarized Top Quark Decay
- Spin Effects in Hadronic $t\bar{t}$ Production: QCD and Weak Corrections
- Summary
Properties of Top Quark

\[ m_t \sim 173 \text{ GeV} \]

Within SM

- Top Decays mainly into: \( t \rightarrow b + W^+ \)
- Top Decay Width: \( \Gamma_t = 1.4 \text{ GeV} \)
  \[ \rightarrow \text{Lifetime} \; \tau_t \sim 4 \times 10^{-25} \text{ sec} < < \text{Characteristic Hadronization time} \sim 28 \times 10^{-25} \text{ sec} \]
  \[ \rightarrow \text{Top Quark Decays before Hadronization!} \]

Top Quark \( \sim \) Quasi-free Instable Particle

1 fm
Properties of Top Quark

\[ t(\bar{t}) \text{ or } \bar{t}t \text{ are Produced in a Specific Spin Configuration} \]

\[ (\text{Depending on the Production Dynamics}) \]

- \[ t \rightarrow b + W^+, \bar{t} \rightarrow \bar{b} + W^-, W \rightarrow l\nu_l, q_1\bar{q}_2 \]
  - Top Spin Information is transferred to the Decay Products
  - Polarization of \( t, \bar{t} \) and \( t\bar{t} \) Spin Correlations are Good Observables
  - Top Quark Spin Phenomena are Measurable

Top Quark Spin Effects

- Reliably Calculable
- Suited to Experimentally Check Predictions of SM or its extensions

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1-loop Calculations for \( t\bar{t} \) Pair Production at Hadron Colliders
Properties of Top Quark

- Top Spin Effects are Useful Tools!
- Analyses Require Precise SM Predictions!!

Our aim: Finish 1-loop Calculations to $\bar{t}t$ Production

- NLO QCD Corrections
- Mixed QCD and EW corrections at 1-loop level:
  Spin Averaged results: Scharf’s talk
  Spin Dependent part
Decay of Polarized $t \rightarrow b + W^+$ in SM:

1. $t \rightarrow W^+ (h_W = -1)$ Allowed: $\text{Prob.} \sim 30\%$.

   ![Diagram](image1)

2. $t \rightarrow W^+ (h_W = 0)$ Allowed: $\text{Prob.} \sim 70\%$.

   ![Diagram](image2)

3. $t \rightarrow W^+ (h_W = +1)$ Forbidden for $m_b = 0$

   ![Diagram](image3)

   non-zero $m_b + \text{QCD} + \text{EW Corr.} \rightarrow \text{Prob.} \sim 0.1\%$. Do et al. (2003)

Important Observables for Determining the Structure of $tbW$-vertex!
Polarized Top Quark Decay

Ensemble of Top Quarks Self-Analyses its Spin Polarization via its Parity-Violating Weak Decays

\[ t \rightarrow W^+ + b \rightarrow \begin{cases} l + \nu_l + b \\ q_1 + \bar{q}_2 + b \end{cases} \]

Standard V-A Charged Current Interaction → Charged Lepton

\( l = e, \mu, \tau \) is the Best Analyzer of Top Spin

Decay Distribution of (100%) Polarized \( t \rightarrow f + \cdots \)

\[
\frac{1}{\Gamma} \frac{d\Gamma}{d\cos\theta_f} = \frac{1}{2} \left( 1 + \kappa_f \cos\theta_f \right)
\]

\( \kappa_l = 1 \) (Maximal)

\( \kappa_b = -\kappa_W = -0.41 \)
Polarized Top Quark Decay

Order $\alpha_s$ Corrections

Semi-leptonic Decays: $t \rightarrow bl^+\nu_l, bl^+\nu_l + g$

Non-leptonic Decays: $t \rightarrow bq_1\bar{q}_2, bq_1\bar{q}_2 + g$
respectively, $t \rightarrow j_bj_1j_2, j_bj_1j_2j_3$

Spin Analyzer Quality Factor $\kappa_f$: 

Czarnecki, Jezabek, Kühn ’91 (semileptonic)
Brandenburg, Si, Uwer ’02 (non-leptonic)

<table>
<thead>
<tr>
<th></th>
<th>$l^+$</th>
<th>$\bar{d}$</th>
<th>$u$</th>
<th>$b$</th>
<th>$j_&lt;$</th>
<th>$j_&gt;$</th>
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<tr>
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<td>-0.31</td>
<td>-0.39</td>
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</tbody>
</table>

$j_<$: Least energetic non-$b$-jet (Durham Algorithm)

$j_>$: most energetic non-$b$-jet (Durham Algorithm)
Spin Effects in Hadronic $t \bar{t}$ Production

Main Reactions

$$ p\bar{p}/pp \rightarrow t\bar{t}X \rightarrow \begin{cases} 2l + n \geq 2\text{jets} + P_T^{\text{miss}} \\ l + n \geq 4\text{jets} + P_T^{\text{miss}} \\ n \geq 6\text{jets} \end{cases} $$

Within SM:

- $t\bar{t}$ Production Dominated by Strong Interactions:
  
  $$ q\bar{q} \rightarrow t\bar{t}, \quad gg \rightarrow t\bar{t}, \ldots $$

- Weak Decays of $t$ and $\bar{t}$ into Semi-leptonic $t \rightarrow b\ell\nu$ and Non-leptonic $t \rightarrow bq_1\bar{q}_2$ Channels
Spin Effects in Hadronic $t\bar{t}$ Production

**Theory: QCD Corrections**

- Spin-averaged Cross Section for $\sigma(pp/p\bar{p} \rightarrow t\bar{t}X)$ Known to order $\alpha_s^3$ + Resummation of ‘Infrared and Threshold Logarithms’
  - Nason et al.; Beenakker et al.
  - Bonciani et al.; Kidonakis, Vogt; Cacciari et al.

- NLO MC Generators for $t\bar{t}$ Production:
  - MC@NLO, MCFM, ...

- NLO QCD Predictions including $t, \bar{t}$ Spin Informations:

  $$q\bar{q} \xrightarrow{t\bar{t}} b + \bar{b} + 4f (+g),$$

  $$gg \xrightarrow{t\bar{t}} b + \bar{b} + 4f (+g),$$

  $$gq(\bar{q}) \xrightarrow{t\bar{t}} b + \bar{b} + 4f + q(\bar{q}), \quad f = q, l, \nu.$$
Spin Effects in Hadronic $t\bar{t}$ Production

Theory: EW Corrections

1. $q\bar{q} \rightarrow t\bar{t}(g)$
   - Born QCD: Gluon exchange
   - Born EW: $\gamma$ and $Z$ exchange
   - $O(\alpha_W\alpha_s^2)$: initial/final vertex corrections
     - Beenakker et al., Kao et al., Beccaria et al.
   - Extensions ($q\bar{q}$): box-contributions + real gluon radiation:
     - Bernreuther, Fücker, Si; Kühn, Scharf, Uwer

2. $gg \rightarrow t\bar{t}$:
   - Moretti et al.; Bernreuther, Fücker, Si; Kühn, Scharf, Uwer
   - Born QCD: Gluon exchange, $t$- and $u$-chanel top exchange
   - $O(\alpha_s^2\alpha)$: Interference of IR finite virtual corrections with Born QCD.
Unstable $t, \bar{t}$ are Narrow Resonances: $\Gamma_t \sim 1.5 \text{GeV} \ll m_t$

- Leading Pole Approximation Appropriate
  (Considering Top as Signal)
- 2 Types of Radiative Corrections:
  Factorizable and Non-Factorizable
Spin Effects in Hadronic $t\bar{t}$ Production

Differential Parton Cross Sections

$$d\sigma_i = d\sigma^0_i + d\sigma_{i,\text{fact.}} + d\sigma_{i,\text{nf}}, \quad i = q\bar{q}, gg, gq, g\bar{q}$$

- Non-factorizable Corr.: mainly from Semi-soft Gluons
  Beenakker, Berends, Chapovsky (1999), L. Meyer (2005)
- Factorizable Corr.: Apply Narrow Width Approximation for $t, \bar{t}$

$$|\mathcal{M}_{\text{fact.}}|^2 \sim \text{Tr}\{ R\rho_t \rho_{\bar{t}} \}$$

1. **Spin-Density Matrix** for Processes:
   $$q\bar{q}, gg, qg, \bar{q}g \to t\bar{t}X$$ to Order $\alpha_s^3$

2. **Decay Density Matrix** for $t \to f_tX$ and $\bar{t} \to f_{\bar{t}}X$ to Order $\alpha_s$

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Technique and Method

1. Feynman-Diagram **virtual** and **real** Corrections:

   - Computer-Algebra
     - Regularize all **Divergences** in \( d=4-2\varepsilon \) Dimensions
     - “Box-integrals”: Calculated in \( d=6 \) Dimensions
       \[ \Rightarrow \text{Box-integral finite} \]
     - Real Correction: **Helicity amplitudes for massive particle**
     - UV-Divergences are removed by **Renormalization**
       \[ \alpha_s: \overline{MS}\text{-Scheme by Scale } \mu_R; m_t: \text{on-shell-Scheme} \]

2. Definition and Calculation for “**collinear safe**” Spin-Observables

   - **Factorization** of **infrared Singularities** (Gluon-energy \( \rightarrow 0 \)) and **collinear Singularity** (Gluon || massless Parton):
     - Remove the Divergences by **Renormalizing the Parton-Distribution-Function** at Factorization Scale \( \mu_F \)

3. Numerical Calculations: **Monte-Carlo Integration**
Spin-Effects in hadronic $t\bar{t}$ Production

Possible Spin-Effects

1. Polarization of $t, \bar{t}$: (very) Small
   - Normal to Production Plane (P-even, T-odd) due to QCD Absorptive Parts
   - Polarization in Production Plane (Parity-violation) due to Weak Interactions

2. $t\bar{t}$ Spin Correlations:
   - Large Effect in SM, mainly due to QCD
   - Strength Depends on the Choice of Reference Axes $\rightarrow$ $t, \bar{t}$ Spin Quantization Axes in On-shell Approximation
Spin-Correlation: Qualitative Analyse

$q\bar{q} \rightarrow t\bar{t}$:

1. **Production Threshold** ($\beta_t \rightarrow 0$): $t\bar{t}$ in $^3S_1$ State
   - $t\bar{t}$-Spins **100% correlated** w. r. t. Beam Basis

2. **High Energy Limit** ($\beta_t \rightarrow 1$): Top-Polarization $||$ Flying-Direction
   - $t\bar{t}$-Spins **100% correlated** w. r. t. Helicity basis
   (helicity conservation of quark gluon inter.)

3. "Off-Diagonal Basis" (Mahlon, Parke)

   \[
   \hat{d} = \frac{-\hat{p} + (1-\gamma)(\hat{p} \cdot \hat{k}_t)\hat{k}_t}{\sqrt{1-(\hat{p} \cdot \hat{k}_t)^2(1-\gamma^2)}}, \quad \gamma = E_t/m_t \quad \Rightarrow <4(\hat{S}_t \cdot \hat{d})(\hat{S}_{\bar{t}} \cdot \hat{d})> = 1 \text{ (LO)}
   \]

$gg \rightarrow t\bar{t}$

Production Threshold: $t\bar{t}$ in $^1S_0$ State
No **Off-Diagonal Basis** exists to produce 100% $t\bar{t}$ correlations!!!
Spin Effects in Hadronic $t\bar{t}$ Production

**$t\bar{t}$ Spin Correlations**

W.R.T Arbitrary Reference Axes $\hat{a}$, $\hat{b}$:

$$\langle 4(\hat{a} \cdot \hat{s}_t)(\hat{b} \cdot \hat{s}_{\bar{t}}) \rangle = A$$

where $A$ is the $t\bar{t}$ Double Spin Asymmetry

$$A = \frac{N(\uparrow\uparrow) + N(\downarrow\downarrow) - N(\uparrow\downarrow) - N(\downarrow\uparrow)}{N(\uparrow\uparrow) + N(\downarrow\downarrow) + N(\uparrow\downarrow) + N(\downarrow\uparrow)}$$

For on-shell $t$, $\bar{t}$: $\hat{a}$, $\hat{b} \leftrightarrow$ Spin Axes:

- $\hat{a} = \hat{k}_t$, $\hat{b} = \hat{k}_{\bar{t}}$ \hspace{1cm} (helicity basis)
- $\hat{a} = \hat{b} = \hat{p}$ \hspace{1cm} (beam basis)
- $\hat{a} = \hat{b} = \hat{d}$ \hspace{1cm} (off – diagonal basis)
Spin Effects in Hadronic $t\bar{t}$ Production

Spin Axes

- At Tevatron, $q\bar{q} \rightarrow t\bar{t} + X$ is the dominant process
  beam basis essentially as good as off-diagonal basis
- at LHC, $gg \rightarrow t\bar{t} + X$ is the dominant process
  helicity basis is the best choice

Consider, e.g., dilepton channels

$$pp, p\bar{p} \rightarrow t\bar{t}X \rightarrow l^+ l^- X$$

$$\int d\sigma = \sum_{ij} \int dx_1 dx_2 f_i^{h_1}(x_1, \mu_F) f_j^{h_2}(x_2, \mu_F)$$

$$\times [d\Phi_6 | M_6 |^2_{LO+NLO} + d\Phi_7 | M_7 |^2_{NLO}]$$
Spin Effects in Hadronic $t\bar{t}$ Production

Double Distribution

$$\frac{1}{\sigma} \frac{d^2\sigma}{d\cos\theta_+ \cos\theta_-} = \frac{1}{4} \left[ 1 + B_1 \cos\theta_+ + B_2 \cos\theta_- - C \cos\theta_+ \cos\theta_- \right]$$

$\theta_+ = \langle \hat{a}_1, \hat{a} \rangle$, $\theta_- = \langle \hat{a}_2, \hat{b} \rangle$, $\hat{a}, \hat{b}$: Spin-Quantization Axes

1. $B_1$ and $B_2$ reflects top quark spin polarization
   - pure QCD effects: component normal to scattering plane
   - Weak int. leads to a component parallel to scattering plane

2. $C$ reflects spin-spin correlations between $t$ and $\bar{t}$
   - contr. from initial $q\bar{q}$ and $gg$ induced by pure QCD effects have different sign $\implies C$ can be used as a tool to determine PDF
   - all-order formula(factorizable corrections):

$$C = \kappa_+ \kappa_- A, \quad -1 \leq C \leq 1$$
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Spin Effects in Hadronic $\bar{t}t$ Production

### PDF Input: CTEQ6L and CTEQ6.1M

<table>
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<th>Tevatron, $\sqrt{s} = 1.96$ GeV</th>
<th>LHC, $\sqrt{s} = 14$ GeV</th>
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<td></td>
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<tr>
<td>$l + l$</td>
<td></td>
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<tr>
<td>$C_{hel}$</td>
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<td>$C_{beam}$</td>
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<tr>
<td>$C_{off}$</td>
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<tr>
<td>$l + j$</td>
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<tr>
<td>$C_{hel}$</td>
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<td>$C_{off}$</td>
<td>0.478</td>
<td>0.372</td>
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Spin Effects in Hadronic $t\bar{t}$ Production

EW Corrections

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<th>$\mu = m_t$</th>
<th>$\mu = 2m_t$</th>
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<td>Tevatron (pb)</td>
<td>NLO QCD</td>
<td>7.493</td>
<td>7.105</td>
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<tr>
<td></td>
<td>Weak</td>
<td>0.0339</td>
<td>0.0355</td>
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<tr>
<td>LHC (pb)</td>
<td>NLO QCD</td>
<td>868.150</td>
<td>850.385</td>
</tr>
<tr>
<td></td>
<td>Weak</td>
<td>-14.127</td>
<td>-10.790</td>
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- EW contributions to the total cross section:
  -1.3% at LHC, 0.5% at Tevatron

- smaller than the scale uncertainties of the fixed-order NLO QCD corrections
PV Spin Asymmetries

Weak Interactions \( \longrightarrow \) P-violating spin asymmetries
eps. \( t, \bar{t} \) polarization in production plane

\[
\langle s_t \cdot \hat{p} \rangle, \quad \langle s_t \cdot \hat{k}_t \rangle, \quad \langle s_{\bar{t}} \cdot \hat{p} \rangle, \quad \langle s_{\bar{t}} \cdot \hat{k}_{\bar{t}} \rangle
\]

- leptonic asymmetries in \( ll \) and \( l + j \) channels:

\[
pp, p\bar{p} \rightarrow t\bar{t}X \rightarrow l^+ + X
\]

\[
\frac{1}{\sigma} \frac{d^2\sigma}{d\cos\theta_+ d\cos\theta_-} = \frac{1}{4} [1 + B_1 \cos\theta_+ + B_2 \cos\theta_- - C \cos\theta_+ \cos\theta_-]
\]

\[
\theta_+ = \angle(\hat{a}_1, \hat{a}), \quad \theta_- = \angle(\hat{a}_2, \hat{b})
\]

\( \hat{a}, \hat{b} \): interpreted as Spin-Quantization Axes
Angular distributions:

\[ \frac{1}{\sigma} \frac{d\sigma}{d \cos \theta_+} = \frac{1}{2} \left[ 1 + B_+ \cos(\theta_+) \right], \quad \theta_+ = \angle(\hat{i}^+, \hat{a}) \]

e.g., \( \hat{a} = \) beam axis (Tevatron), helicity basis (LHC)

\[ A_{PV} = B_+ = \frac{N(\uparrow) - N(\downarrow)}{N(\uparrow) + N(\downarrow)} \]

**Table:** SM prediction for the parity-violating asymmetry for Tevatron and LHC.

<table>
<thead>
<tr>
<th>( M_{t \bar{t}}^* ) [GeV]</th>
<th>( A_{PV} ), Tevatron</th>
<th>( M_{t \bar{t}}^* ) [GeV]</th>
<th>( A_{PV} ), LHC</th>
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<tr>
<td>400</td>
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<td>500</td>
<td>0.0056</td>
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<td>1000</td>
<td>-0.0052</td>
<td>1500</td>
<td>0.022</td>
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Within SM, P-Violating top polarizations very small

- To be investigated: with which precision \( A_{PV} \) can actually be measured at LHC?
Summary

**$t\bar{t}$ production:**
- spin correlations, $t$ and $\bar{t}$ polarization: SM predictions at NLO (QCD and EW) are available.
- Within SM, PV top polarizations are small.
- more on non-SM studies should be done · · ·

**Top-quark spin physics**
- important tools to explore the dynamics of top quarks.
- remains to be fully explored.

**Thanks a lot for your attention!**