1-loop Calculations for $t\bar{t}$ Pair Production at Hadron Colliders

Zong-Guo SI

Department of Physics, Shandong University

16-18 Apri, LoopVestVI, Fermilab

Outline

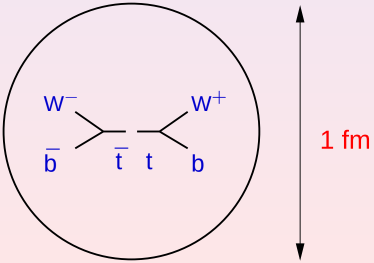
- Properties of Top Quark
- Polarized Top Quark Decay
- Spin Effects in Hadronic $t\bar{t}$ Production:
  QCD and Weak Corrections
- Summary

Properties of Top Quark

$m_t \sim 173$ GeV

Within SM

- Top Decays mainly into: $t \longrightarrow b + W^+$
- Top Decay Width: $\Gamma_t = 1.4$ GeV
  $\longrightarrow$ Lifetime $\tau_t \sim 4 \times 10^{-25}$sec $<<$ Characteristic Hadronization time $\sim 28 \times 10^{-25}$sec
  $\longrightarrow$ Top Quark Decays before Hadronization!



Top Quark $\sim$ Quasi-free Instable Particle

Properties of Top Quark

$t(\bar{t})$ or $t\bar{t}$ are Produced in a Specific Spin Configuration (Depending on the Production Dynamics)

- $t \longrightarrow b + W^+$, $\bar{t} \longrightarrow \bar{b} + W^-$, $W \longrightarrow l\nu_l$, $q_1\bar{q}_2$
  Top Spin Information is transfered to the Decay Products
  $\Rightarrow$ Polarization of t, $\bar{t}$ and $t\bar{t}$ Spin Correlations are Good Observables
  $\Rightarrow$ Top Quark Spin Phenomena are Measurable

Top Quark Spin Effects

- Reliably Calculable
- Suited to Experimentally Check Predictions of SM or its extensions

Properties of Top Quark

- Top Spin Effects are Useful Tools!
- **Analyses Require Precise SM Predictions!!**

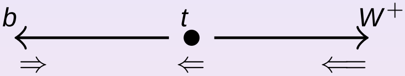
Our aim: Finish 1-loop Calculations to $t\bar{t}$ Production

- **NLO QCD Corrections**
- **Mixed QCD and EW corrections at 1-loop level:**
  Spin Averaged results: Scharf's talk
  Spin Dependent part

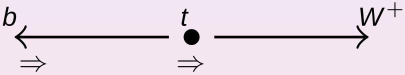
Polarized Top Quark Decay

Decay of Polarized $t \to b + W^+$ in SM:

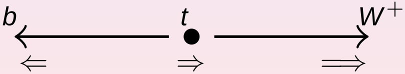
1. $t \to W^+(h_W = -1)$ Allowed: $Prob. \sim 30\%$.



2. $t \to W^+(h_W = 0)$ Allowed: $Prob. \sim 70\%$.



3. $t \to W^+(h_W = +1)$ Forbidden for $m_b = 0$



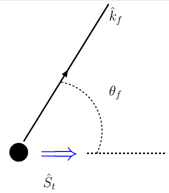
non-zero m_b+QCD+EW Corr. $\to Prob. \sim 0.1\%$. Do et al. (2003)

Important Observables for Determining the Structure of *tbW*-vertex!

Ensemble of Top Quarks Self-Analyses its Spin Polarization via its Parity-Violating Weak Decays

$$t \longrightarrow W^+ + b \longrightarrow \left[\begin{array}{l} l + \nu_l + b \\ q_1 + \bar{q}_2 + b \end{array} \right.$$

Standard V-A Charged Current Interaction $\longrightarrow$ Charged Lepton $l = e, \mu, \tau$ is the Best Analyzer of Top Spin



Decay Distribution of (100%) Polarized $t \longrightarrow f + \cdots$

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\vartheta_f} = \frac{1}{2}\left(1 + \kappa_f \cos\vartheta_f\right)$$

$\kappa_l = 1$ (Maximal)

$\kappa_b = -\kappa_W = -0.41$

Polarized Top Quark Decay

Order α_s Corrections

Semi-leptonic Decays: $t \longrightarrow bl^+\nu_l,\, bl^+\nu_l + g$

Non-leptonic Decays: $t \longrightarrow bq_1\bar{q}_2,\, bq_1\bar{q}_2 + g$
respectively, $t \longrightarrow j_b j_1 j_2,\, j_b j_1 j_2 j_3$

Spin Analyzer Quality Factor κ_f:

Czarnecki, Jezabek, Kühn '91 (semileptonic)
Brandenburg, Si, Uwer '02 (non-leptonic)

	l^+	$\bar{d}$	u	b	$j_<$	$j_>$
LO:	1	1	-0.32	-0.41	0.51	0.2
NLO:	0.999	0.966	-0.31	-0.39	0.47	

$j_<$: Least energetic non-b-jet(Durham Algorithm)
$j_>$: most energetic non-b-jet(Durham Algorithm)

Main Reactions

$$p\bar{p}/pp \longrightarrow t\,\bar{t}\,X \longrightarrow \begin{cases} 2l + n \geq 2jets + P_T^{miss} \\ l + n \geq 4jets + P_T^{miss} \\ n \geq 6jets \end{cases}$$

Within SM:

- $t\bar{t}$ Production Dominated by Strong Interactions:

$$q\bar{q} \longrightarrow t\bar{t}, \quad gg \longrightarrow t\bar{t}, \cdots$$

- Weak Decays of t and $\bar{t}$ into Semi-leptonic $t \longrightarrow bl\nu_l$ and Non-leptonic $t \longrightarrow bq_1\bar{q}_2$ Channels

Theory: QCD Corrections

- Spin-averaged Cross Section for $\sigma(pp/p\bar{p} \to t\bar{t}X)$ Known to order α_s^3 + Resummation of 'Infared and Threshold Logrithms'
  Nason et al.; Beenakker et al.
  Bonciani et al.; Kidonakis, Vogt; Cacciari et al.
- NLO MC Generators for $t\bar{t}$ Production:
  MC@NLO, MCFM, $\cdots$
- NLO QCD Predictions including t, $\bar{t}$ Spin Informations:

$$q\bar{q} \xrightarrow{t\bar{t}} b + \bar{b} + 4f\,(+g),$$
$$gg \xrightarrow{t\bar{t}} b + \bar{b} + 4f\,(+g),$$
$$gq(\bar{q}) \xrightarrow{t\bar{t}} b + \bar{b} + 4f + q(\bar{q}), \quad f = q, l, \nu.$$

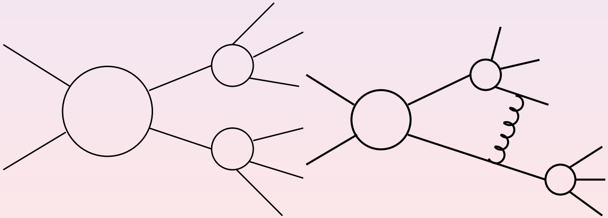
Theory: EW Corrections

1. $q\bar{q} \longrightarrow t\bar{t}(g)$
   - Born QCD: Gluon exchange
   - Born EW: γ and Z exchange
   - $\mathcal{O}(\alpha_W \alpha_s^2)$: initial/final vertex corrections
     Beenakker et al., Kao et al., Beccaria et al.
   - Extensions($q\bar{q}$): box-contributions + real gluon radiation:
     Bernreuther, Fücker, Si; Kühn, Scharf, Uwer
2. $gg \longrightarrow t\bar{t}$:
   Moretti et al.; Bernreuther, Fücker, Si; Kühn, Scharf, Uwer
   - Born QCD: Gluon exchange, t- and u-chanel top exchange
   - $O(\alpha_s^2\alpha)$: Interference of IR finite virtual corrections with Born QCD.

Spin Effects in Hadronic $t\bar{t}$ Production

Unstable t, $\bar{t}$ are Narrow Resonances: $\Gamma_t \sim 1.5 GeV << m_t$

- Leading Pole Approximation Aprropriate (Considering Top as Signal)
- 2 Types of Radiative Corrections: Factorizable and Non-Factorizable



Spin Effects in Hadronic $t\bar{t}$ Production

Differential Parton Cross Sections

$$d\sigma_i = d\sigma_i^0 + d\sigma_{i,fact.} + d\sigma_{i,nf}, \quad i = q\bar{q}, gg, gq, g\bar{q}$$

- Non-factorizable Corr.: mainly from Semi-soft Gluons
  Beenakker, Berends, Chapovsky (1999), L. Meyer (2005)
- Factorizable Corr.: Apply Narrow Width Approximation for t, $\bar{t}$

$$\left|\mathcal{M}_{fact.}\right|^2 \sim Tr\left\{R\, \rho_t\, \rho_{\bar{t}}\right\}$$

1. R **Spin-Density Matrix** for Processes:
   $q\bar{q}, gg, qg, \bar{q}g \longrightarrow t\bar{t}X$ to Order α_s^3
2. $\rho_t, \rho_{\bar{t}}$ **Decay Density Matrix** for $t \longrightarrow f_t X$ and $\bar{t} \longrightarrow f_{\bar{t}} X$ to Order α_s

Technique and Method

1. Feynman-Diagram **virtual** and **real** Corrections:
   Computer-Algebra
   - Regularize all **Divergences** in **d=4-2ε** Dimensions
   - "Box-integrals": Calculated in **d=6** Dimensions
     $\Rightarrow$ Box-integral finite
   - Real Correction: **Helicity amplitudes for massive particle**
   - UV-Divergences are removed by **Renormalization**
     α_s: $\overline{MS}$-Scheme by Scale μ_R; m_t: on-shell-Scheme
2. Definition and Calculation for **"collinear safe" Spin-Observables**
   - **Factorization** of **infrared Singularities** (Gluon-energy $\longrightarrow$ 0) and **collinear Singularity** (Gluon $||$ massless Parton):
   - Remove the Divergences by **Renormalizing the Parton-Distribution-Function** at Factorization Scale μ_F
3. Numerical Calculations: Monte-Carlo Integration

Spin-Effects in hadronic $t\bar{t}$ Production

Possible Spin-Effects

1. **Polarization of t,$\bar{t}$: (very) Small**
   - Normal to Production Plane(P-even, T-odd) due to QCD Absorptive Parts
   - Polarization in Production Plane(Parity-violation) due to Weak Interactions

2. **$t\bar{t}$ Spin Correlations:**
   - Large Effect in SM, mainly due to QCD
   - Strength Depends on the Choice of Reference Axes $\longrightarrow$ $t, \bar{t}$ Spin Quantization Axes in On-shell Approximation

Spin-Correlation: Qualitative Analyse

$q\bar{q} \longrightarrow t\bar{t}$:

1. **Production Threshold ($\beta_t \to 0$):** $t\bar{t}$ in $^3S_1$ State
   $\Rightarrow t\bar{t}$-Spins **100% correlated** w. r. t. Beam Basis
2. **High Energy Limit ($\beta_t \to 1$):** Top-Polarization $||$ Flying-Direction
   $\Rightarrow t\bar{t}$-Spins **100%correlated** w. r. t. Helicity basis
   (helicity conservation of quark gluon inter.)
3. "Off-Diagonal Basis" (Mahlon, Parke)

$$\hat{\mathbf{d}} = \frac{-\hat{\mathbf{p}} + (1-\gamma)(\hat{\mathbf{p}}\cdot\hat{\mathbf{k}}_t)\hat{\mathbf{k}}_t}{\sqrt{1-(\hat{\mathbf{p}}\cdot\hat{\mathbf{k}}_t)^2(1-\gamma^2)}}, \quad \gamma = E_t/m_t \quad \Rightarrow < 4(\hat{\mathbf{S}}_t \cdot \hat{\mathbf{d}})(\hat{\mathbf{S}}_{\bar{t}} \cdot \hat{\mathbf{d}} >= 1 \text{ (LO)}$$

$gg \longrightarrow t\bar{t}$

Production Threshold: $t\bar{t}$ in $^1S_0$ State
No Off-Diagonal Basis exists to produce 100% $t\bar{t}$ correlations!!!

$t\bar{t}$ Spin Correlations

W.R.T Arbitrary Reference Axes $\hat{\mathbf{a}}$, $\hat{\mathbf{b}}$:

$$< 4(\hat{\mathbf{a}} \cdot \hat{\mathbf{s}}_t)(\hat{\mathbf{b}} \cdot \hat{\mathbf{s}}_{\bar{t}}) >= A$$

where A is the $t\bar{t}$ Double Spin Asymmetry

$$A = \frac{\mathrm{N}(\uparrow\uparrow) + \mathrm{N}(\downarrow\downarrow) - \mathrm{N}(\uparrow\downarrow) - \mathrm{N}(\downarrow\uparrow)}{\mathrm{N}(\uparrow\uparrow) + \mathrm{N}(\downarrow\downarrow) + \mathrm{N}(\uparrow\downarrow) + \mathrm{N}(\downarrow\uparrow)}$$

For on-shell $t, \bar{t}$: $\hat{\mathbf{a}}$, $\hat{\mathbf{b}} \leftrightarrow$ Spin Axes:

$$\hat{\mathbf{a}} = \hat{\mathbf{k}}_t, \quad \hat{\mathbf{b}} = \hat{\mathbf{k}}_{\bar{t}} \qquad (helicity\ basis)$$

$$\hat{\mathbf{a}} = \hat{\mathbf{b}} = \hat{\mathbf{p}} \qquad (beam\ basis)$$

$$\hat{\mathbf{a}} = \hat{\mathbf{b}} = \hat{\mathbf{d}} \qquad (off-diagonal\ basis)$$

Spin Axes

- At Tevatron, $q\bar{q} \to t\bar{t} + X$ is the dominant process
  beam basis essentially as good as off-diagonal basis
- at LHC, $gg \to t\bar{t} + X$ is the dominant process
  helicity basis is the best choice

Consider, e.g., dilepton channels

$$pp, p\bar{p} \to t\bar{t}X \to l^+ l'^- X$$

$$\int d\sigma = \sum_{ij} \int dx_1 dx_2 f_i^{h_1}(x_1, \mu_F) f_j^{h_2}(x_2, \mu_F) \times [d\Phi_6 |M_6|^2_{LO+NLO} + d\Phi_7 |M_7|^2_{NLO}]$$

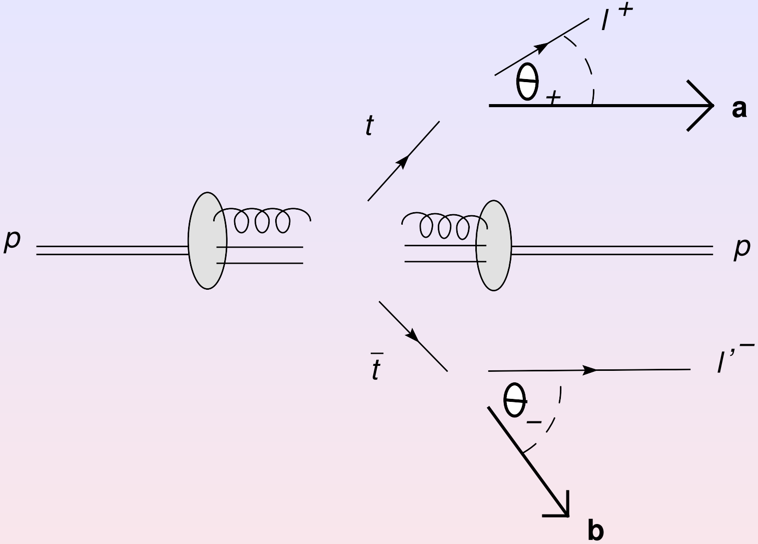
Double Distribution

$$\frac{1}{\sigma}\frac{d^2\sigma}{d\cos\theta_+\ \cos\theta_-} = \frac{1}{4}[1 + B_1\ \cos\theta_+ + B_2\ \cos\theta_- - C\ \ \cos\theta_+\ \cos\theta_-]$$

$\theta_+ = \sphericalangle(\hat{\mathbf{a}}_1, \hat{\mathbf{a}})$, $\theta_- = \sphericalangle(\hat{\mathbf{a}}_2, \hat{\mathbf{b}})$, $\hat{\mathbf{a}}, \hat{\mathbf{b}}$: **Spin-Quantization Axes**

1. B_1 and B_2 reflects top quark spin polarization
   - pure QCD effects: component normal to scattering plane
   - Weak int. leads to a component parallel to scattering plane
2. C reflects spin-spin correlations between t and $\bar{t}$
   - contr. from initial $q\bar{q}$ and gg induced by pure QCD effects have different sign $\Longrightarrow$ C can be used as a tool to determine PDF
   - all-order formula(factorizable corrections):

$$C = \kappa_+\kappa_- A, \qquad -1 \le C \le 1$$



l +
Θ +
a
t
p
p
t̄
l'−
Θ −
b

Spin Effects in Hadronic $t\bar{t}$ Production

PDF Input: CTEQ6L and CTEQ6.1M

	Tevatron, $\sqrt{s} = 1.96$ GeV		LHC, $\sqrt{s} = 14$ GeV	
$l + l$	LO	NLO	LO	NLO
C_{hel}	-0.471	-0.352	0.319	0.326
C_{beam}	0.928	0.777	-0.005	-0.072
C_{off}	0.937	0.782	-0.027	-0.089
$l + j$				
C_{hel}	-0.240	-0.168	0.163	0.158
C_{beam}	0.474	0.370		
C_{off}	0.478	0.372		

Bernreuther, Brandenburg, Si, Uwer, NPB690(2004)81.

EW Corrections

Total cross section($\sigma_{t\bar{t}}$) from QCD and weak interactions

		$\mu = m_t/2$	$\mu = m_t$	$\mu = 2m_t$
Tevatron (pb)	NLO QCD	7.493	7.105	6.314
	Weak	0.0339	0.0355	0.0346
LHC (pb)	NLO QCD	868.150	850.385	793.543
	Weak	−14.127	−10.790	−8.368

Bernreuther, Fücker, Si, PRD74(2006)113005.

- EW contributions to the total cross section:
  -1.3% at LHC, 0.5% at Tevatron
- smaller than the scale uncertainties of the fixed-order NLO QCD corrections

PV Spin Asymmetries

Weak Interactions $\longrightarrow$ P-violating spin asymmetries

eps. t, $\bar{t}$ polarization in production plane

$$< \mathbf{s}_t \cdot \hat{\mathbf{p}} >, \quad < \mathbf{s}_t \cdot \hat{\mathbf{k}}_t >, \quad < \mathbf{s}_{\bar{t}} \cdot \hat{\mathbf{p}} >, \quad < \mathbf{s}_{\bar{t}} \cdot \hat{\mathbf{k}}_{\bar{t}} >$$

- leptonic asymmetries in ll and $l+j$ channels:

$$pp, p\bar{p} \rightarrow t\bar{t}X \rightarrow l^+ + X$$

$$\frac{1}{\sigma}\frac{d^2\sigma}{d\cos\theta_+ \ \cos\theta_-} = \frac{1}{4}[1 + B_1 \ \cos\theta_+ + B_2 \ \cos\theta_- - C \ \ \cos\theta_+ \ \cos\theta_-]$$

$$\theta_+ = \sphericalangle(\hat{\mathbf{a}}_1, \hat{\mathbf{a}}), \quad \theta_- = \sphericalangle(\hat{\mathbf{a}}_2, \hat{\mathbf{b}})$$

$\hat{\mathbf{a}}, \hat{\mathbf{b}}$: interpreted as **Spin-Quantization Axes**

Angular distributions:

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta_+} = \frac{1}{2}\left[1 + B_+\cos(\theta_+)\right], \quad \theta_+ = \sphericalangle(\hat{\mathbf{l}}^+, \hat{\mathbf{a}})$$

e.g., $\hat{\mathbf{a}}$ = beam axis(Tevatron), helicity basis(LHC)

$$A_{PV} = B_+ = \frac{N(\uparrow) - N(\downarrow)}{N(\uparrow) + N(\downarrow)}$$

Table: SM prediction for the parity-violating asymmetry for Tevatron and LHC.

$M^*_{t\bar{t}}$ [GeV]	A_{PV}, Tevatron	$M^*_{t\bar{t}}$ [GeV]	A_{PV}, LHC
400	-0.0054	500	0.0056
700	-0.0060	1000	0.0154
1000	-0.0052	1500	0.022

Within SM, P-Violating top polarizations very small

- To be investigated: with which precision A_{PV} can actually be measured at LHC?

Summary

$t\bar{t}$ production:

- spin correlations, t and $\bar{t}$ polarization:
  SM predictions at NLO (QCD and EW) are available
- Within SM, PV top polarizations are small
- more on non-SM studies should be done $\cdots$

Top-quark spin physics

- important tools to explore the dynamics of top quarks
- remains to be fully explored

Thanks a lot for your attention!