NLO QCD corrections to tri-boson production

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Loopfest VI, April 16, 2007, Fermilab

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Outline

- Introduction
- Method
- Results
- Conclusions

Introduction

- NLO QCD computations are required for a large number of multiparticle processes that will be important at the LHC.
- It was repeatedly stated in the past that existing methods for performing such calculations are not up to the task.
- Indeed, compare the speed with which processes disappear from the NLO wishlist with the speed at which “new techniques for NLO computations” appear.
- I will describe yet another “new” method and will apply it to compute $pp \to 3Z$ through NLO QCD.
- The method I want to suggest:
  - is fully numerical;
  - can be easily automated;
  - works for massive and massless internal/external particles;
  - does not require any reductions of tensor integrals.

Method

- Numerical NLO computations are difficult because Feynman integrals are divergent.
- Different types of singularities: ultraviolet, soft, collinear, threshold; Feynman parameters are useful for isolating them.

$$I_N \sim \Gamma(N - d/2) \int \Pi \mathrm{d}x_i \frac{\delta(1 - \sum x_i)\mathrm{Num}(\{x_i\})}{(x_i S_{ij} x_j - i\delta)^{N-d/2}}.$$

- Soft and collinear singularities $\Leftrightarrow$ singular $x_i \to 0$ limits, after primary sector decompozition.
- Threshold singularities $\Leftrightarrow$ quadratic form $x_i S_{ij} x_j$ vanishes for $x_i \neq 0$.
- It is fairly easy to take care of the soft/collinear and threshold singularities separately:
  - soft/collinear: sector decompozition; Binoth, Heinrich
  - threshold: contour deformation. Soper
- The two techniques can be combined producing a fairly general method for numerical NLO computations.

Method

- Sector decompozition is an easy-to-automate procedure that allows algorithmic extraction of infrared/collinear singularities from loop integrals.
- Consider the following integral

$$I = \int_0^1 \mathrm{d}x\mathrm{d}y \frac{x^{-\epsilon}}{(x+y)^2}.$$

Split the integration region into $x < y$ and $y < x$. In the first region, do $x \to xy$; in the second region $y \to xy$. Then,

$$I = \int_0^1 \mathrm{d}x\mathrm{d}y \; \frac{x^{-\epsilon} y^{-\epsilon-1}}{(1+x)^2} + \int_0^1 \mathrm{d}x\mathrm{d}y \; \frac{x^{-\epsilon-1}}{(1+y)^2}.$$

- Extract singularities using the plus-distribution prescription

$$x^{n\epsilon-1} = \frac{1}{n\epsilon}\delta(x) + \sum_{i=0}^{\infty} \frac{(n\epsilon)^i}{i!} \left[\frac{\ln^i(x)}{x}\right]_+ .$$

Method

- Sector decompozition takes care of the soft/collinear singularities; however, threshold singularities are still unsuitable for numerical integration.
- Soper suggested to avoid threshold singularities by deforming integration contour:
  - after primary sector decompozition, the quadratic form in the denominator of a Feynman integral becomes

$$\Delta = x_i S_{ij} x_j - i\delta \Rightarrow t_i X_{ij} t_j + t_i Y_i + Z - i\delta, \quad 0 \le t_i \le 1;$$

  - change $t_i \Rightarrow z_i = t_i - i\lambda t_i(1-t_i)f_i$, where $f_i = 2X_{ij}t_j + Y_i$.
    It is constructed in such a way that $\mathrm{Im}(\Delta) < 0$, along the integration path.
  - Applied recently to n-photon scattering through massive electron loop.

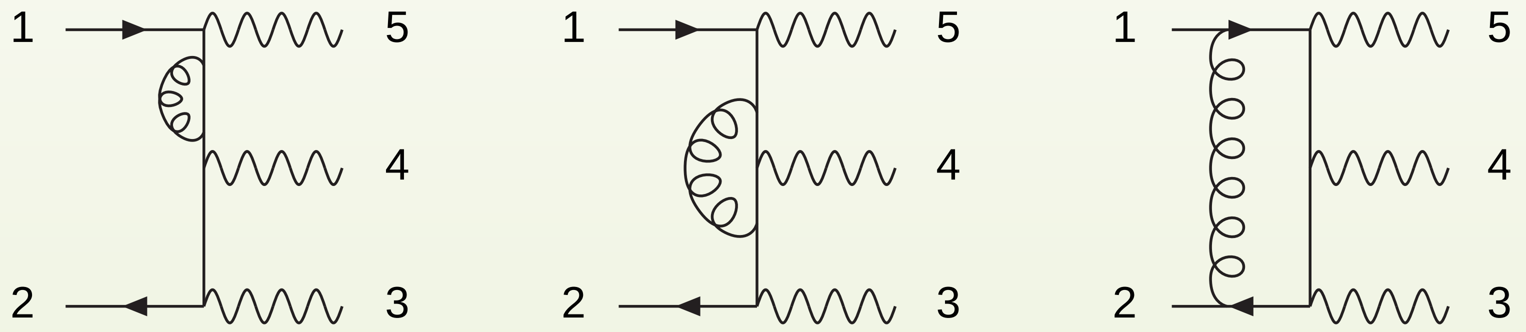
Soper, Nagy

Method

- The two techniques can be combined; we get a tool for numerical computations of Feynman diagrams with arbitrary singularity structure.
- The sector decompozition should be performed first.
- After the sector decompozition, the polynomial in the denominator is not a quadratic form anymore; it becomes a higher degree polynomial.
- This is not a problem but choosing appropriate contour deformation and making sure that no poles in the complex plane are crossed becomes a bit of an issue.
- No need for a special treatment of tensor integrals.

$pp \to 3Z$

- We apply these techniques to $pp \to 3Z$, one of the processes from the “wishlist”.
- As usual, at NLO physical results are obtained from a sum of real emission and virtual corrections.
- Real emission corrections are well-understood.
- Virtual corrections



- Good process to test the method:
  - relatively few diagrams (~ 50 for NLO virtual);
  - just one channel $q\bar{q} \to 3Z$ for virtual corrections;
  - but five-point functions are present, so non-trivial test.

$pp \to 3Z$

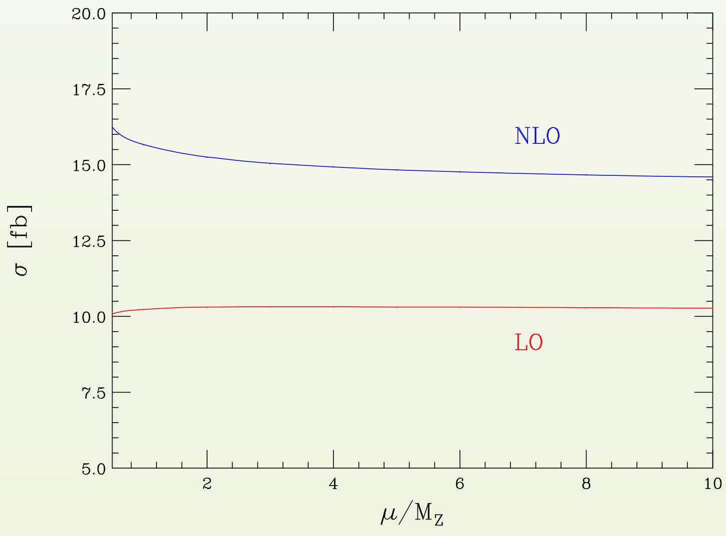
- We obtain the NLO virtual corrections for 10^4 randomly generated phase-space points; requires few days of running on a cluster that consists of a few dozens PCs.
- Excellent numerical stability throughout the phase-space; the NLO virtual corrections are computed with sub-percent precision.
- Self-energies, vertices and boxes are computed very fast and very accurate.
- For five-point functions, judicious choice of Feynman parameters is crucial for stable numerical evaluation.
- The efficiency of numerical integration depends strongly on λ, the size of the contour deformation.
- We do not let Z bosons decay but this should not be a problem.

$pp \to 3Z$: results

- Total cross-sections: $\sigma_{\rm LO} = 10.3$ fb, $\sigma_{\rm NLO} = 15.7$ fb for $\mu = 3\ M_Z$.
- NLO QCD corrections are $\sim 50\%$; a coherent effect of the NLO QCD virtual effects and the LO→NLO change in parton distribution functions.
- Similar large corrections appear in $pp \to VV$.

  Ohnemus, Owens, Mele, Nason, Ridolfi

- Insignificant dependence of $\sigma(pp \to 3Z)$ on the factorization and renormalization scales.

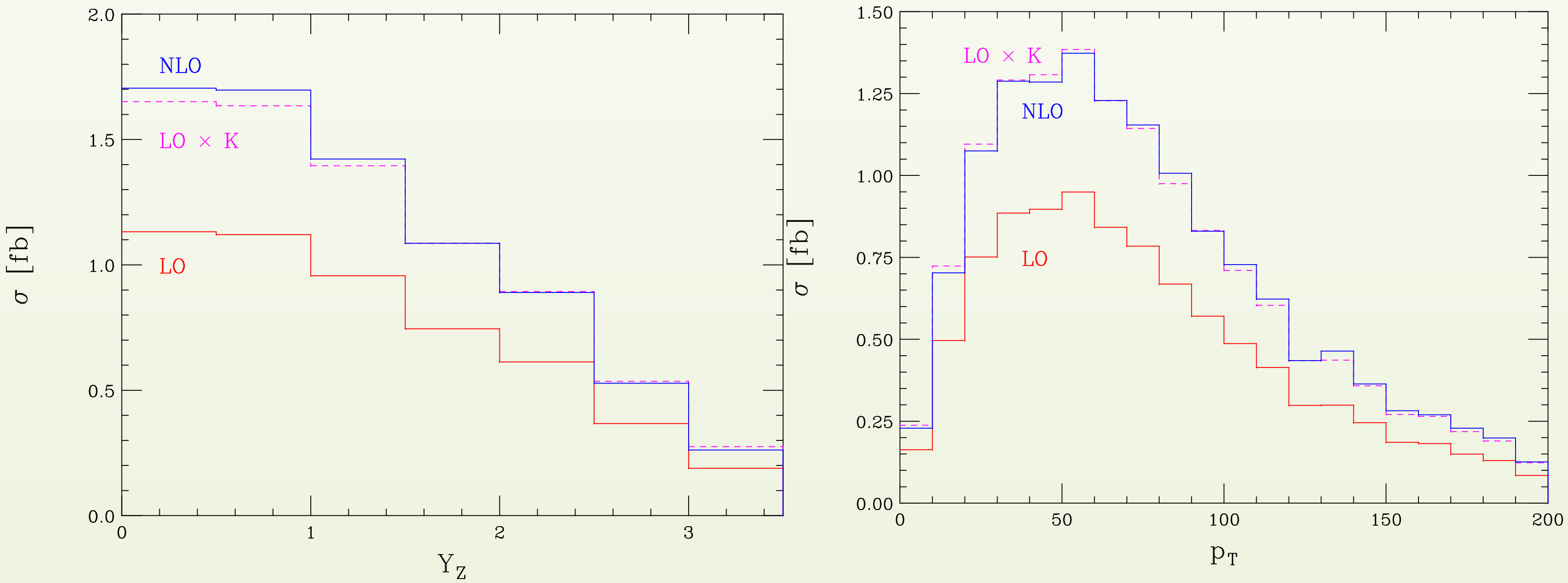


- The leading order result is “scale-independent” and underestimates the actual size of the NLO QCD corrections.

$pp \to 3Z$: results

- Rapidity and transverse momentum distribution of Z-bosons ($\mu = 3M_Z$).

$$Y_z = \frac{1}{3}\sum_i^3 Y^{(i)}, \quad p_\perp = \frac{1}{3}\sum_i^3 p_\perp^{(i)}.$$



- NLO QCD corrections do not change shapes of these distributions.

Conclusions

- I discussed a novel approach to NLO computations that combines sector decompozition with the integration contour deformation.
- Attractive features:
  - very general, easy-to-automate fully numerical method;
  - works for massive/massless internal/external particles;
  - shows good numerical stability;
  - no obvious obstacles for higher-loop applications. Anastasiou, Daleo
- Verified reliability of the method on $pp \to 3Z$.
- Large $\sim 50\%$ NLO QCD corrections, similar to $pp \to VV$.
- Scale dependence of the LO result underestimates the NLO corrections dramatically.
- For basic observables NLO QCD effects do not change shapes of distributions and $\mathrm{d}\sigma_{\mathrm{LO}} \otimes K$ approximation is sufficient.