NLO QCD corrections to tri-boson production

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Introduction

- NLO QCD computations are required for a large number of multiparticle processes that will be important at the LHC.
- It was repeatedly stated in the past that existing methods for performing such calculations are not up to the task.
- Indeed, compare the speed with which processes disappear from the NLO wishlist with the speed at which “new techniques for NLO computations” appear.
- I will describe yet another “new” method and will apply it to compute $pp \rightarrow 3Z$ through NLO QCD.
- The method I want to suggest:
  - is fully numerical;
  - can be easily automated;
  - works for massive and massless internal/external particles;
  - does not require any reductions of tensor integrals.
Method

- Numerical NLO computations are difficult because Feynman integrals are divergent.

- Different types of singularities: ultraviolet, soft, collinear, threshold; Feynman parameters are useful for isolating them.

\[ I_N \sim \Gamma(N - d/2) \int \Pi dx_i \frac{\delta(1 - \sum x_i) \text{Num}(\{x_i\})}{(x_i S_{ij} x_j - i\delta)^{N-d/2}}. \]

- Soft and collinear singularities ⇔ singular \( x_i \to 0 \) limits, after primary sector decomposition.

- Threshold singularities ⇔ quadratic form \( x_i S_{ij} x_j \) vanishes for \( x_i \neq 0 \).

- It is fairly easy to take care of the soft/collinear and threshold singularities separately:
  - soft/collinear: sector decomposition; Binoth, Heinrich
  - threshold: contour deformation. Soper

- The two techniques can be combined producing a fairly general method for numerical NLO computations.
Method

- Sector decomposition is an easy-to-automate procedure that allows algorithmic extraction of infrared/collinear singularities from loop integrals.

- Consider the following integral

\[
I = \int_0^1 \int_0^1 \frac{x^{-\epsilon}}{(x + y)^2} \, dx \, dy.
\]

Split the integration region into \( x < y \) and \( y < x \). In the first region, do \( x \to xy \); in the second region \( y \to xy \). Then,

\[
I = \int_0^1 \int_0^{1/(1+x)} \frac{x^{-\epsilon} y^{-\epsilon-1}}{(1+x)^2} \, dx \, dy + \int_0^1 \int_0^{1/(1+y)} \frac{x^{-\epsilon-1}}{(1+y)^2} \, dx \, dy.
\]

- Extract singularities using the plus-distribution prescription

\[
x^{n\epsilon-1} = \frac{1}{n\epsilon} \delta(x) + \sum_{i=0}^{\infty} \frac{(n\epsilon)^i}{i!} \left[ \frac{\ln^i(x)}{x} \right]_+.
\]
Method

- Sector decomposition takes care of the soft/collinear singularities; however, threshold singularities are still unsuitable for numerical integration.
- Soper suggested to avoid threshold singularities by deforming integration contour:
  - after primary sector decomposition, the quadratic form in the denominator of a Feynman integral becomes
    \[
    \Delta = x_i S_{ij} x_j - i\delta \Rightarrow t_i X_{ij} t_j + t_i Y_i + Z - i\delta, \quad 0 \leq t_i \leq 1;
    \]
  - change \( t_i \Rightarrow z_i = t_i - i\lambda t_i (1 - t_i) f_i \), where \( f_i = 2X_{ij} t_j + Y_i \).
    It is constructed in such a way that \( \text{Im}(\Delta) < 0 \), along the integration path.
  - Applied recently to \( n \)-photon scattering through massive electron loop.

*Soper, Nagy*
Method

• The two techniques can be combined; we get a tool for numerical computations of Feynman diagrams with arbitrary singularity structure.

• The sector decomposition should be performed first.

• After the sector decomposition, the polynomial in the denominator is not a quadratic form anymore; it becomes a higher degree polynomial.

• This is not a problem but choosing appropriate contour deformation and making sure that no poles in the complex plane are crossed becomes a bit of an issue.

• No need for a special treatment of tensor integrals.
We apply these techniques to $pp \to 3Z$, one of the processes from the “wishlist”.

As usual, at NLO physical results are obtained from a sum of real emission and virtual corrections.

Real emission corrections are well-understood.

Virtual corrections

- Good process to test the method:
  - relatively few diagrams ($\sim 50$ for NLO virtual);
  - just one channel $q\bar{q} \to 3Z$ for virtual corrections;
  - but five-point functions are present, so non-trivial test.
We obtain the NLO virtual corrections for $10^4$ randomly generated phase-space points; requires few days of running on a cluster that consists of a few dozens PCs.

- Excellent numerical stability throughout the phase-space; the NLO virtual corrections are computed with sub-percent precision.

- Self-energies, vertices and boxes are computed very fast and very accurate.

- For five-point functions, judicious choice of Feynman parameters is crucial for stable numerical evaluation.

- The efficiency of numerical integration depends strongly on $\lambda$, the size of the contour deformation.

- We do not let $Z$ bosons decay but this should not be a problem.
**pp → 3Z: results**

- Total cross-sections: $\sigma_{\text{LO}} = 10.3 \text{ fb}$, $\sigma_{\text{NLO}} = 15.7 \text{ fb}$ for $\mu = 3 \, M_Z$.
- NLO QCD corrections are $\sim 50\%$; a coherent effect of the NLO QCD virtual effects and the LO→NLO change in parton distribution functions.
- Similar large corrections appear in $pp \rightarrow VV$.
- Insignificant dependence of $\sigma(pp \rightarrow 3Z)$ on the factorization and renormalization scales.

![Graph showing NLO and LO cross-sections](image)

- The leading order result is “scale-independent” and underestimates the actual size of the NLO QCD corrections.
$pp \rightarrow 3Z$: results

- Rapidity and transverse momentum distribution of $Z$-bosons ($\mu = 3M_Z$).

$$Y_z = \frac{1}{3} \sum_i 3 Y^{(i)}, \quad p_\perp = \frac{1}{3} \sum_i 3 p^{(i)}_\perp.$$  

- NLO QCD corrections do not change shapes of these distributions.
Conclusions

- I discussed a novel approach to NLO computations that combines sector decomposition with the integration contour deformation.

- Attractive features:
  - very general, easy-to-automate fully numerical method;
  - works for massive/massless internal/external particles;
  - shows good numerical stability;
  - no obvious obstacles for higher-loop applications.

- Verified reliability of the method on $pp \rightarrow 3Z$.

- Large $\sim 50\%$ NLO QCD corrections, similar to $pp \rightarrow VV$.

- Scale dependence of the LO result underestimates the NLO corrections dramatically.

- For basic observables NLO QCD effects do not change shapes of distributions and $d\sigma_{LO} \otimes K$ approximation is sufficient.