Refined physical masses in supersymmetry

Loopfest VI

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I will report on refined calculations of the gluino, squark and Higgs masses in the MSSM beyond leading order.

Based in part on:

`hep-ph/0701051`

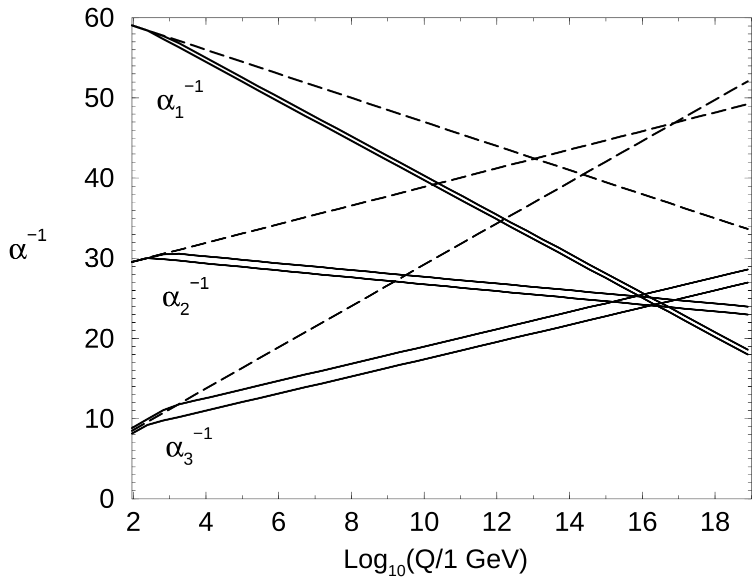
`hep-ph/0608026`

`hep-ph/0501132` (`TSIL`) with Dave Robertson

Masses are key observables for deciphering SUSY breaking

Most of what we do not already know about supersymmetric extensions of the Standard Model involves the soft SUSY-breaking terms with positive mass dimension.

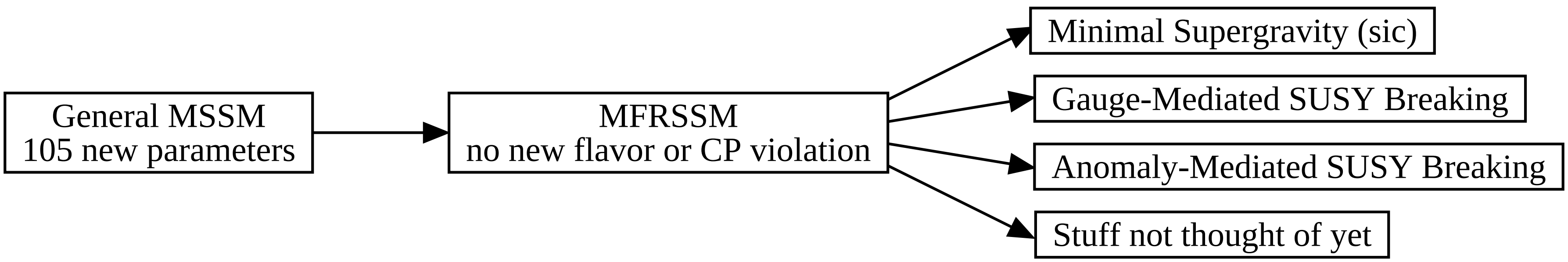
Predictions of specific models (Minimal Supergravity, Gauge Mediation, Anomaly Mediation, Extra-dimensional Mediation, ...) allow/require precise calculations.



The apparent unification of gauge couplings in the MSSM invites us to extrapolate the soft masses up to high scales, to see if they obey some Organizing Principle.

What is the Organizing Principle behind SUSY breaking?

A reasonable working hypothesis is the **Minimal Flavor-Respecting Supersymmetric Standard Model**. It is neither too painfully general, nor too naively specific:



MFRSSM parameter count:

3 gaugino masses	$M_1,\ M_2,\ M_3$
5 sfermion $(\text{mass})^2$	$m^2_{\tilde{Q}},\ m^2_{\tilde{u}},\ m^2_{\tilde{d}},\ m^2_{\tilde{L}},\ m^2_{\tilde{e}}$
3 $(\text{scalar})^3$ couplings	$A_{u0},\ A_{d0},\ A_{e0}$
3 Higgs mass parameters	$\mu,\ b,\ m^2_{H_u},\ m^2_{H_d}$ (but M_Z known)
1 input RG scale	Q_0

Total: 15 new parameters beyond the Standard Model

Gaugino Mass Unification is a popular and recurring theme.

$$M_1(Q) = M_2(Q) = M_3(Q) \equiv m_{1/2} \qquad \text{at } Q \approx 2 \times 10^{16} \text{ GeV},$$

resulting in

$$M_1 : M_2 : M_3 \;\approx\; 1 : 2 : 6$$

for Q near the TeV scale.

To test this, or alternatives to it, we have to relate physical masses to running masses in the Lagrangian (with no superpartners decoupled).

Goal: reduce purely theoretical sources of uncertainty to a negligible level, if possible.

Key features of the approach:

- To test high-scale organizing principles, work in non-decoupled SUSY theory with $\overline{\text{DR}}'$ (mass-independent) renormalization scheme.
- Two-loop diagrams relevant for SUSY involve many comparable but distinct mass scales simultaneously.
- Mass orderings and hierarchies are difficult to anticipate in advance. So don't try.
- Methods should be generic and reusable.

To calculate physical masses

Evaluate self-energy = sum of 1-particle irreducible Feynman diagrams:

$$\Pi(s) = \Pi^{(1)}(s) + \Pi^{(2)}(s) + \ldots$$

where $s =$ the external momentum invariant.

The complex pole mass

$$s_{\rm pole} = M^2 - i\Gamma M$$

is the solution for complex s of:

$$\begin{aligned} s_{\rm pole} &= m^2_{\rm tree} + \Pi(s_{\rm pole}) \\ &= m^2_{\rm tree} + \Pi^{(1)}(m^2_{\rm tree})\left[1 + \Pi^{(1)\prime}(m^2_{\rm tree})\right] + \Pi^{(2)}(m^2_{\rm tree}) + \ldots \end{aligned}$$

The pole mass is gauge invariant at each order in perturbation theory, can be related to kinematic masses as measured at colliders.

In the MSSM, squarks, charginos, neutralinos, Higgs scalars can mix:

$$\Pi_j{}^k(s) = \Pi_j^{(1)k}(s) + \Pi_j^{(2)k}(s) + \ldots$$

Define functions of the $\overline{\text{DR}}$ masses and couplings:

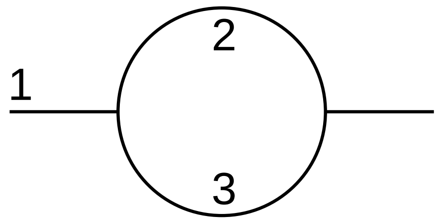
$$\begin{aligned}
\widetilde{\Pi}_j^{(1)k} &= \lim_{s \to m_j^2 + i\epsilon} \Pi_j^{(1)k}(s) \\
\widetilde{\Pi}_j^{(2)k} &= \lim_{s \to m_j^2 + i\epsilon} \left[\Pi_j^{(2)k} + \Pi_j^{(1)k} \frac{\partial}{\partial s} \Pi_j^{(1)k} \right]
\end{aligned}$$

Then the complex pole mass is a gauge-invariant observable:

$$M_j^2 - i\Gamma_j M_j = m_j^2 + \widetilde{\Pi}_j^{(1)k} + \left[\widetilde{\Pi}_j^{(2)k} + \sum_{k \neq j} \widetilde{\Pi}_j^{(1)k} \widetilde{\Pi}_k^{(1)j} / (m_j^2 - m_k^2) \right] + \ldots$$

However, with tree-level masses in the loop integrals, the kinematics can lead to slow convergence of perturbation theory…

For example, the imaginary parts of the one-loop contribution:



will turn on when the $\overline{\text{DR}}$ tree-level masses satisfy:

$$m_1 > m_2 + m_3$$

However, the physical width Γ should actually be non-zero only for physical masses satisfying:

$$M_1 > M_2 + M_3.$$

This can be particularly troublesome for the gluino, squarks and h^0, where the tree-level masses differs greatly from the physical masses. In some cases, the computed Γ in the pole mass can even be negative.

To address this, re-expand the pole mass corrections by defining new functions at each loop order L:

$$\overline{\Pi}_j^{(L)k} = \left\{ \widetilde{\Pi}_j^{(L)k} \text{ with all tree-level masses replaced by the real parts of pole masses} \right\}$$

The pole mass can then be rewritten as:

$$\begin{aligned} M_j^2 - i\Gamma_j M_j \quad &= \quad m_j^2 + \overline{\Pi}_j^{(1)k} + \Bigg[\overline{\Pi}_j^{(2)k} + \sum_{k \neq j} \overline{\Pi}_j^{(1)k} \overline{\Pi}_k^{(1)j} / (M_j^2 - M_k^2) \\ &\quad - \sum_k \mathrm{Re}[\overline{\Pi}_k^{(1)k}] \frac{\partial}{\partial M_k^2} \overline{\Pi}_j^{(1)j} \Bigg] + \ldots \end{aligned}$$

This is formally equivalent to the previous expression, up to terms of 3-loop order. It is similar to the on-shell mass or pole mass computed in the on-shell scheme, but with all couplings and mixing matrices computed at tree-level in $\overline{\text{DR}}$.

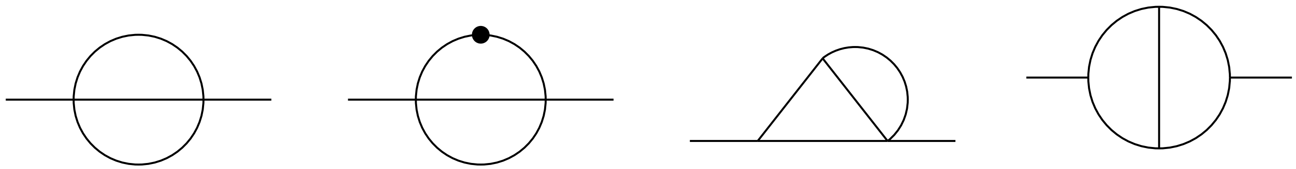
In the following, I will refer to the first method as “expansion around tree masses”, and the method just given as “expansion around pole masses”.

Method:

- Reduce all self-energies in general theory to a few basis integrals

- Numerically evaluate basis integrals quickly and reliably for arbitrary masses.

Tarasov's basis and recurrence relations:



S T U M ("Master integral")

Can always reduce 2-loop self-energies to a linear combination of these, with coefficients rational functions of:

$s = p^2 =$ external momentum invariant

$x, y, z, \ldots$ = internal propagator masses

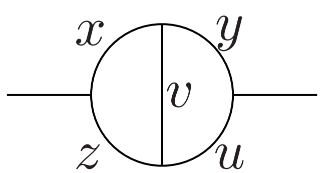
To evaluate basis integrals:

Values at $s = 0$ are known analytically, in terms of logs, polylogs.

$$\frac{\partial}{\partial s}\text{(basis integral)} = \text{(another self-energy integral)}$$
$$= \text{(linear combination of basis integrals)}$$

So, we have a set of coupled, first-order, linear differential equations.

Consider the Master integral $M(x, y, z, u, v)$:



and the 12 U, S, T basis integrals obtained from it by removing propagators.

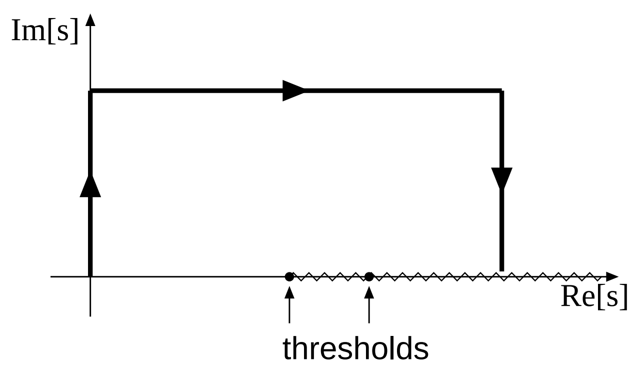
Call these 13 integrals I_n, $(n = 1, \ldots, 13)$.

Differential equations method for basis integrals

$$\frac{d}{ds} I_n = \sum_m K_{nm} I_m + C_n$$

Here K_{nm} are rational functions of s and $x, y, z \ldots$, and C_n are one-loop integrals. These are obtained by using Tarasov's recursion relations.

Solve for basis integrals I_n using Runge-Kutta integration in the complex s-plane, starting from known values at $s = 0$.



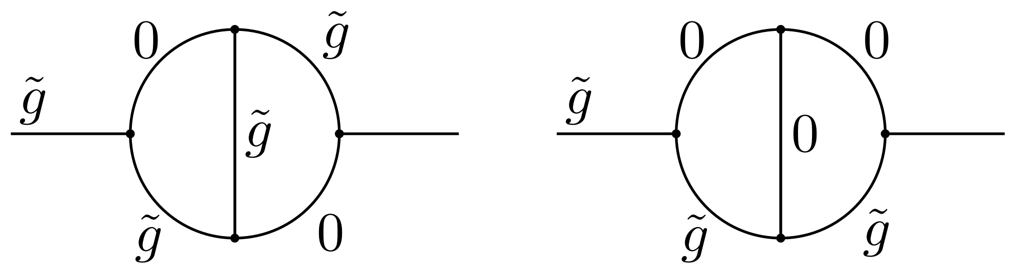
`TSIL` $=$ **T**wo-Loop **S**elf-energy **I**ntegral **L**ibrary

D.G. Robertson, SPM, `hep-ph/0501132`

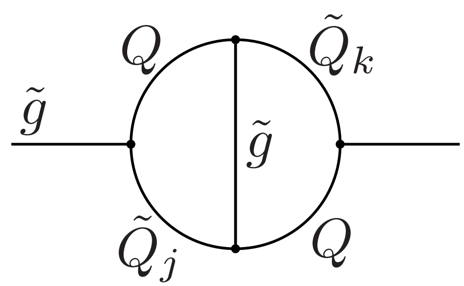
Program written in C, callable from C++, Fortran

- Basis integrals computed for any values of all masses and s.
- All subordinate integrals (S, T, U) for a given master integral (M) are obtained together in a single numerical computation.
- Checks on the numerical accuracy follow from changing choice of contour.
- `TSIL` knows all special cases that have been done analytically in terms of polylogarithms
- Computation times generically $\ll$ 1 second on modern hardware. About 5 to 10 times faster than `s2lse`, typically.

For the SUSYQCD corrections to the gluino ($\tilde{g}$) pole mass, here are the only necessary Master topologies:



(analytic)



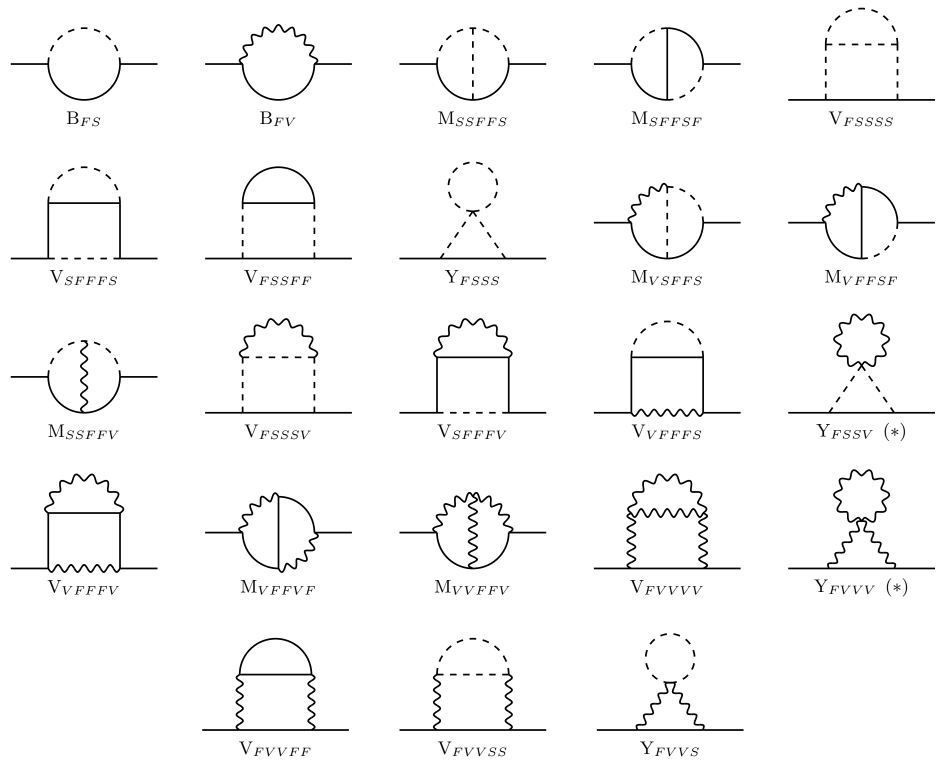
(numerical)

If the first two family squarks are (are not) degenerate, then 12 (18) numerical evaluations by `TSIL` are required, after taking into account the symmetry.

(Note: number of Feynman diagrams is far greater.)

These are the Feynman diagram topologies:

Each diagram can be reduced to integrals calculated as part of the Master integral cases on the previous slide.



+ fermion mass insertions

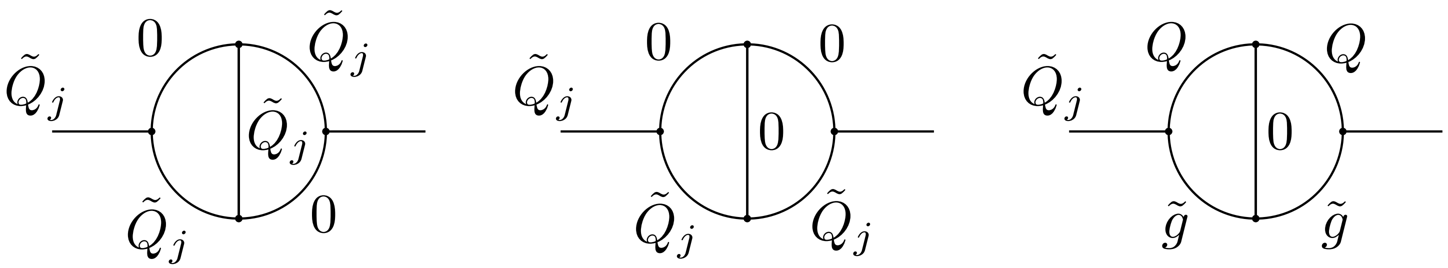
+ ghost diagrams

+ counterterms

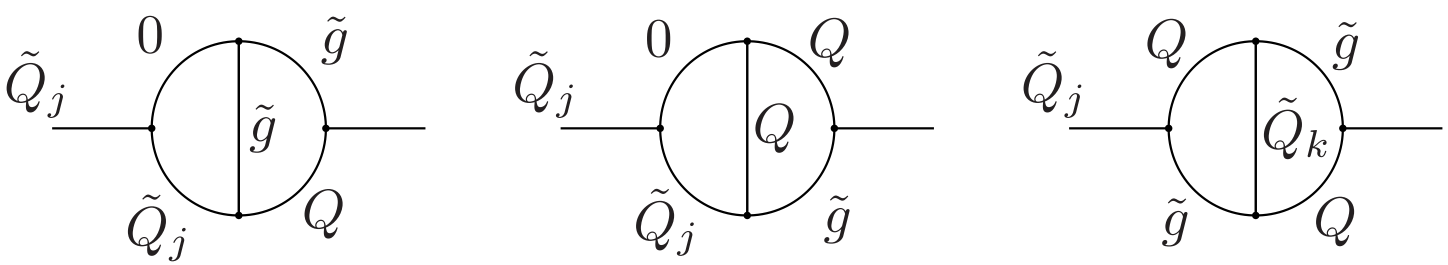
The same integrals have been applied to obtain the SUSYQCD neutralino and chargino pole mass calculation.

(SPM hep-ph/0509115; see also the independent calculation of Robert Schöfbeck, next talk.)

For the SUSYQCD corrections to the squark ($\tilde{Q}_j$) pole mass, the necessary Master topologies are:



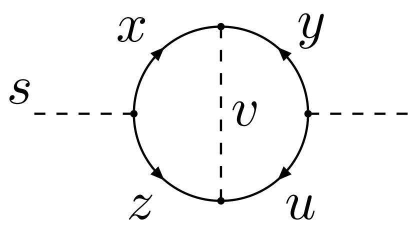
(analytic)



(numerical)

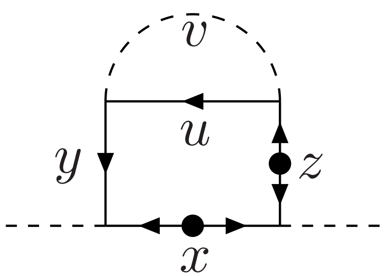
Assuming small flavor violation, each of the 12 squark pole masses requires 3 (4) numerical evaluations of TSIL, if there is (is not) L-R squark mixing.

Examples of reduction to basis integrals:



$$\equiv \quad M_{FFFFS}(x,y,z,u,v)$$

$$\begin{aligned} = \quad & (xu + yz - vs)M(x,y,z,u,v) - xU(z,x,y,v) - zU(x,z,u,v) \\ & -uU(y,u,z,v) - yU(u,y,x,v) + S(x,u,v) + S(y,z,v) \\ & +sB(x,z)B(y,u) \end{aligned}$$



$$\equiv \quad V_{\overline{F}F\overline{F}FS}(x,y,z,u,v)$$

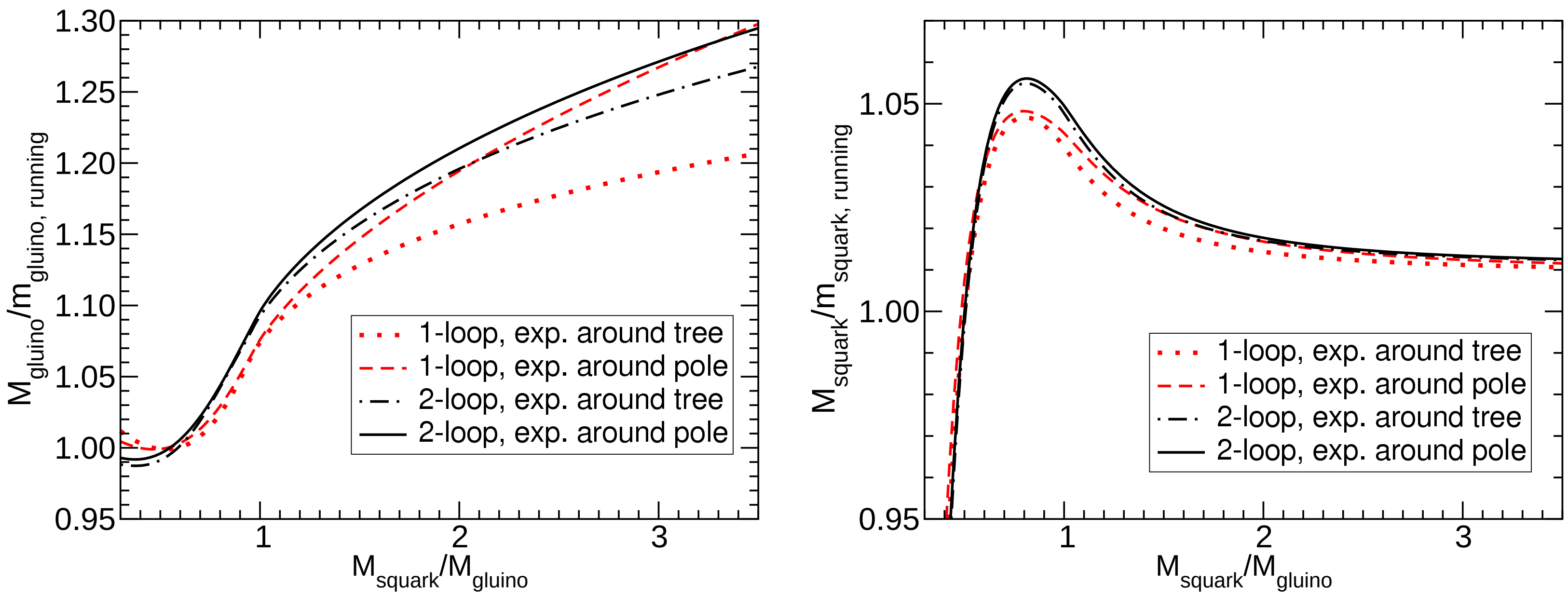
$$= \quad \{(v-y-u)U(x,z,u,v) + [A(v) - A(u)]B(x,y)\}/(y-z) + (y \leftrightarrow z)$$

etc.

The same functions are also applicable to Higgs and slepton self-energies.

Numerical results for gluino and squark pole masses

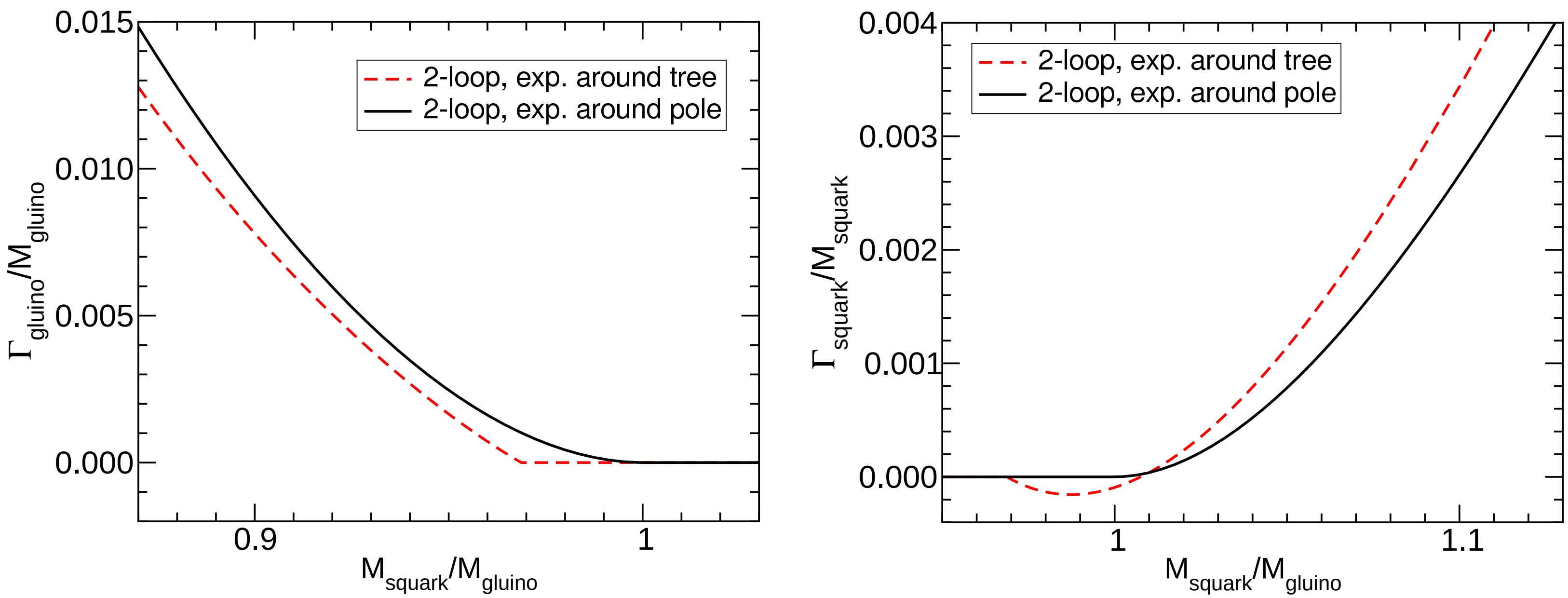
For simplicity of presentation, take all squarks degenerate and unmixed, and all quarks (including top!) massless.



The "expansion around pole" method is probably most accurate.

(The two-loop gluino pole mass in the case of no squark mixing was computed independently and first by Youichi Yamada.)

The widths of the gluino and squark pole masses as extracted from the complex pole squared masses $M^2 - i\Gamma M$:

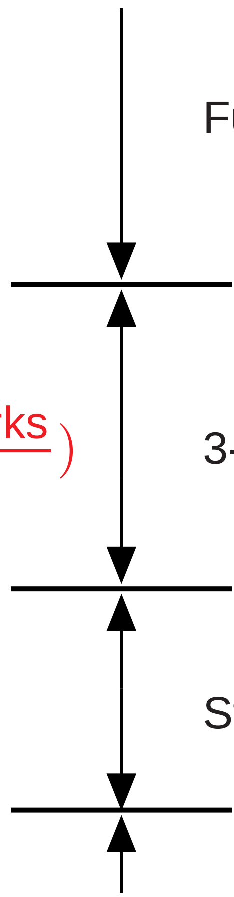


The “expansion around tree” method fails to give a width consistent with kinematics near the threshold region, as remarked earlier.

The “expansion around pole” method is consistent with kinematics, and gives exact agreement with the NLO gluino and squark decay widths found in Beenakker, Hopker, Zerwas `hep-ph/9602378`.

Three-loop gluino mass corrections for heavy squarks

Exploit the fact that beta functions are easier to compute, known to $\geq$3-loop order. Let the running parameters in the full MSSM be α_s, M_3, and in the effective theory with squarks decoupled, $\widehat{\alpha}_s, \widehat{M}_3$.



Full MSSM, no decoupling

M_{squarks} — 2-loop threshold corrections give $(\widehat{\alpha}_s, \widehat{M}_3) \leftrightarrow (\alpha_s, M_3)$

$L = \ln(\frac{M_{\text{squarks}}}{M_3})$ — 3-loop $\widehat{\alpha}_s$ and $\widehat{M}_3$ beta functions known (same as “Split SUSY”)

$M_{\tilde{g}}$ — 3-loop $M_{\tilde{g}}^{\text{pole}}$ known in terms of $\widehat{\alpha}_s$ and $\widehat{M}_3$

Standard Model: 4-loop QCD beta function known

M_{top}

To obtain the 3-loop contributions for large $L = \ln(M_{\tilde{Q}}/M_{\tilde{g}}),$ need:

- **2-loop** threshold corrections for M_3 in MSSM
  (SPM 2006)
- **2-loop** threshold corrections for α_s in MSSM
  (Bern, DeFreitas, Dixon, Wong 2002; Harlander, Mihaila, Steinhauser 2005)
- **2-loop** pole mass in a theory with only fermions
  (Gray, Broadhurst, Grafe, Schilcher 1990)
- **3-loop** mass beta function in a theory with only fermions but in different reps
  (Tarasov 1982, unpublished, available from KEK server, only in Russian!)

In addition, using:

- **3-loop** pole mass in a theory with only fermions (Melnikov and van Ritbergen 1999) but with different reps

I obtain the subset of non-log-enhanced contributions that don't involve heavy particle loops, which I expect will be the most important.

Using the effective field theory matching and RG running technique, one obtains

$\alpha_s^n L^n$	1-loop β functions, 0-loop threshold matching
$\alpha_s^n L^{n-1}$	2-loop β functions, 1-loop threshold matching
$\alpha_s^n L^{n-2}$	3-loop β functions, 2-loop threshold matching

for all $n =$ loop order.

For the "expand around tree" method,

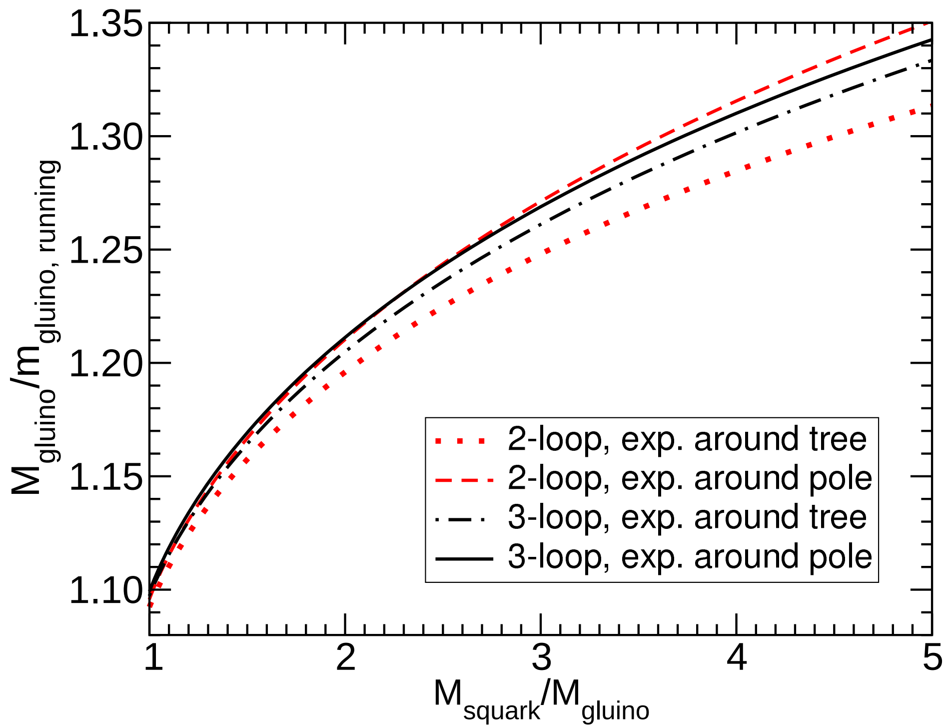
$$\begin{aligned} \Delta_{3-\mathrm{loop}} M_{\tilde{g}}^2, &= \left(\frac{\alpha_S}{4\pi}\right)^3 \left[0.16\ell^3 + 1.11\ell^2 + 3.74\ell + 9.62 + ?\right] m_{\tilde{g}}^2 \\ \ell &= \ln(m_{\tilde{Q}}^2/m_{\tilde{g}}^2). \end{aligned}$$

For the "expand around poles" method,

$$\begin{aligned} \Delta_{3-\mathrm{loop}} M_{\tilde{g}}^2 &= \left(\frac{\alpha_S}{4\pi}\right)^3 \left[0.16L^3 - 1.16L^2 - 2.16L + 5.99 + ?\right] M_{\tilde{g}}^2 \\ L &= \ln(M_{\tilde{Q}}^2/M_{\tilde{g}}^2). \end{aligned}$$

Here, "?" means non-log-enhanced contributions from heavy sparticle loops.

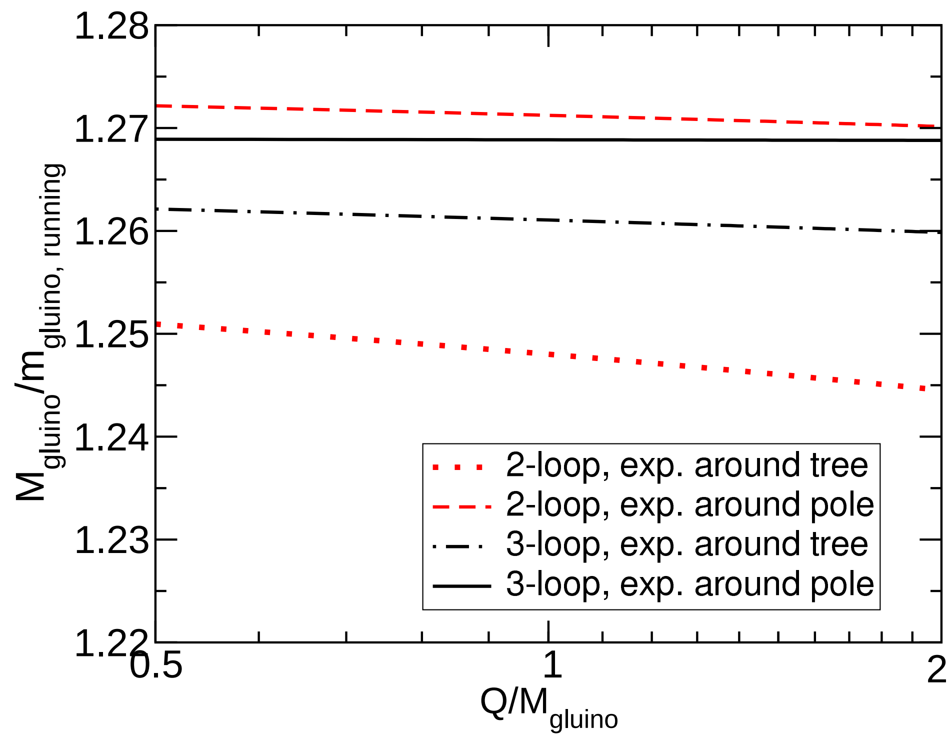
Adding these 3-loop effects to the preceding 2-loop results:



The two three-loop approximations are much closer to each other, as expected.

The difference between the two-loop and three-loop “expand around pole” results is also quite small.

RG scale dependence of various approximations, for $M_{\tilde{Q}}/M_{\tilde{g}} = 3$:



The three-loop leading log approximation is specifically engineered to give a RG scale invariant answer (up to terms of 4-loop order), so this is not a surprise.

In hep-ph/0405022 and earlier papers, I have used the same techniques to find the 2-loop Higgs pole masses in SUSY, including:

- All 1-loop effects
- All 2-loop effects of order:
  - QCD + Yukawa: $\alpha_s y_t^2$, $\alpha_s y_b^2$
  - Yukawa: y_t^4, $y_t^2 y_b^2$, y_b^4, y_τ^4 etc.
  - QCD + Electroweak: $\alpha_s g^2$, $\alpha_s g g'$, $\alpha_s g'^2$.
  - Yukawa + Electroweak: $y_t^2 g^2$, etc.
- All other 2-loop effects in the effective potential approximation. (Taking external momentum $= 0$ in the self-energies.)

In all contributions except the last, the effects of arbitrary MSSM complex phases are included.

To go further, use the renormalization group and effective field theory matching conditions to resum the largest logs.

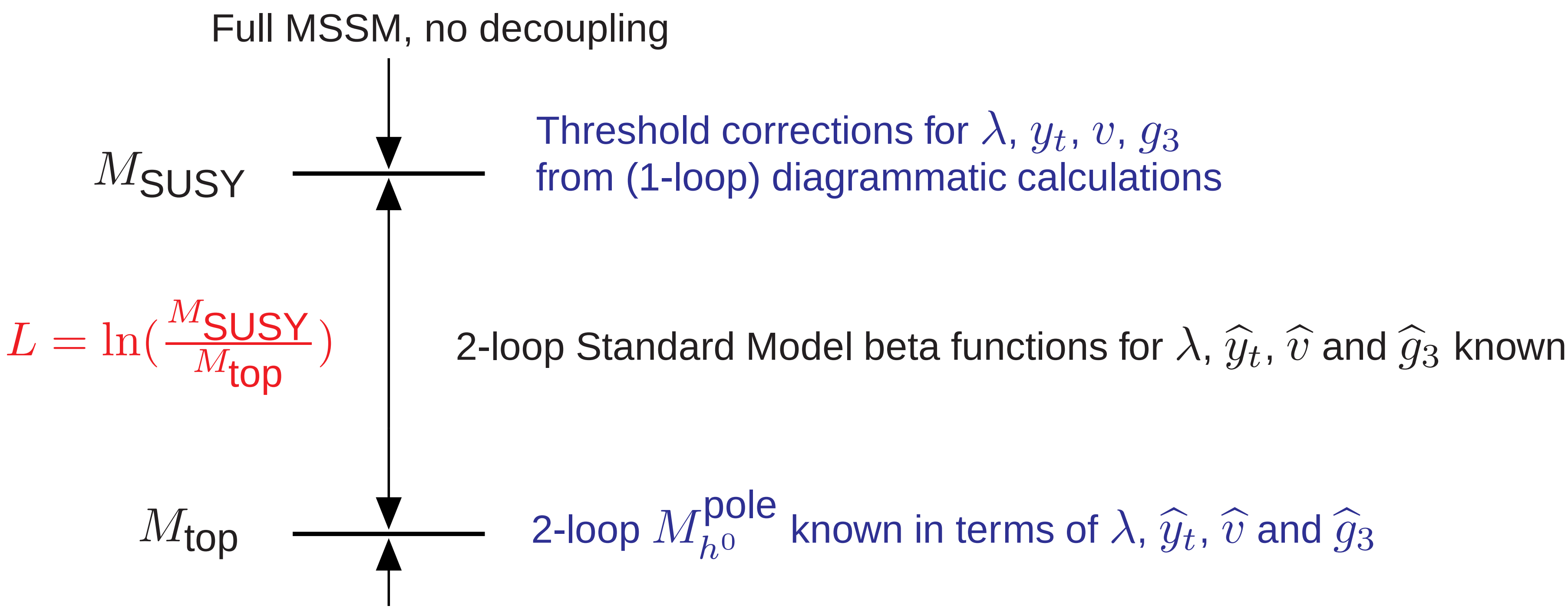
Matching of MSSM ($\overline{\text{DR}}$) to Standard Model ($\overline{\text{MS}}$) for the Higgs coupling, VEV, Yukawa, and g_3:

$$\begin{aligned}
\widehat{\lambda} &= \frac{1}{2}(g^2+g'^2)c_{2\beta}^2 + y_t^4 s_\beta^4 C_\lambda + \ldots, \\
\widehat{v} &= (v_u/s_\beta)\left[1 + y_t^2 s_\beta^2 C_v + \ldots\right], \\
\widehat{y}_t &= y_t s_\beta \left[1 + \left(g_3^2 C_{y_t} + y_t^2 s_\beta^2 C'_{y_t}\right) + \ldots\right], \\
\widehat{g}_3 &= g_3 \left[1 + g_3^2 C_{g_3} + \ldots\right].
\end{aligned}$$

where, for example,

$$\begin{aligned}
C_\lambda &= \frac{3}{8\pi^2}\Big[\ln(m_{\tilde{t}_1}^2/Q^2) + \ln(m_{\tilde{t}_2}^2/Q^2) + 2|X_t|^2 \ln(m_{\tilde{t}_2}^2/m_{\tilde{t}_1}^2)/(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2) \\
&\quad + |X_t|^4 \Big\{2 - [(m_{\tilde{t}_2}^2 + m_{\tilde{t}_1}^2)/(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)] \ln(m_{\tilde{t}_2}^2/m_{\tilde{t}_1}^2)\Big\}/(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)^2\Big] \\
C_{g_3} &= \frac{1}{16\pi^2}\left[\overline{\ln}(m_{\tilde{g}}^2) - \frac{1}{2} + \frac{1}{12}\sum_{i=1}^{12}\overline{\ln}(m_{\tilde{q}_i}^2)\right]
\end{aligned}$$

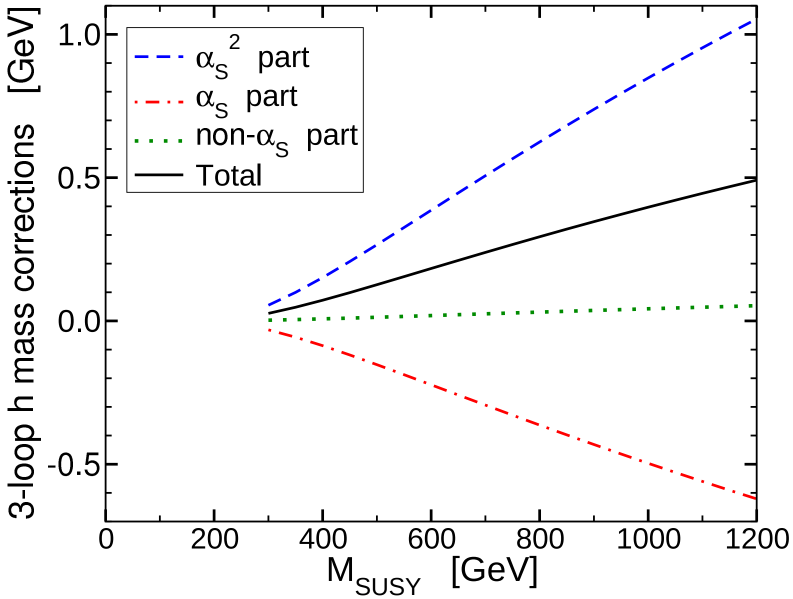
Schematic picture of the strategy:

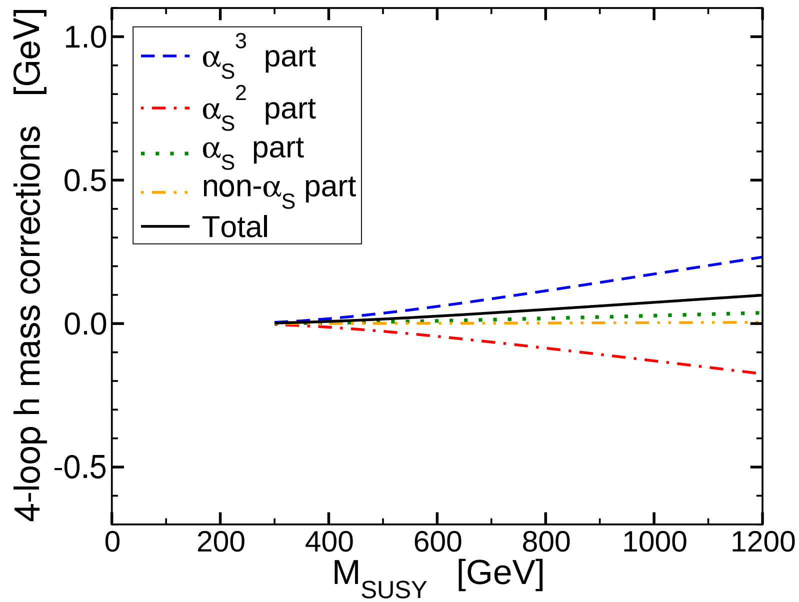


M_{SUSY} is an arbitrary RG scale, typically of order the stop masses.

(Method does not assume degenerate or unmixed superpartners.)

To illustrate the size of effects:





For simplicity, only the leading-log is kept, with $M_h = 120$ GeV, $M_t = 172$ GeV, $\tan\beta \gg 1$, and $m_{\tilde{t}_1} = m_{\tilde{t}_2} = m_{\tilde{g}} = M_{\rm SUSY}$.

These corrections are specific to the scheme and method used for the 1-loop and 2-loop parts; in this case, pure $\overline{\rm DR}'$ masses and couplings everywhere. Comparison with other approaches is non-trivial!

The Future

For practical reasons, the calculations described above need to be implemented in publicly available and open computer programs.

Dave Robertson and I have begun work on a program, `Supermodel` , which will implement the MSSM mass spectrum at two-loop order, with leading-log three-loop contributions for the gluino and lightest Higgs. We hope to have a releasable version around the end of this year.