Antenna Subtraction with Hadronic Initial States

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Motivation

For precise predictions for jet observables

- Efficients methods for loop integrals and real corrections
- Method for real correction
  - Phase space slicing
  - Sector decomposition
  - Subtraction method

Subtraction Method

NLO differential partonic correction

$$
\begin{aligned}
d\sigma^{\text{NLO}} &= d\sigma_m^{\text{V+CT}} + d\sigma_{m+1}^{\text{R}} \\
&= (d\sigma_m^{\text{V+CT}} + d\sigma_{m+1}^{\text{R,S}}) + (d\sigma_{m+1}^{\text{R}} - d\sigma_{m+1}^{\text{R,S}})
\end{aligned}
$$

The subtraction term $d\sigma_{m+1}^{\text{R,S}}$

- Reproduces the behavior of the ME in all singular limits
- Can be integrated analytically over the unresolved phase space
- Does not introduce spurious singularities

Antenna Subtraction

Many methods available for NLO

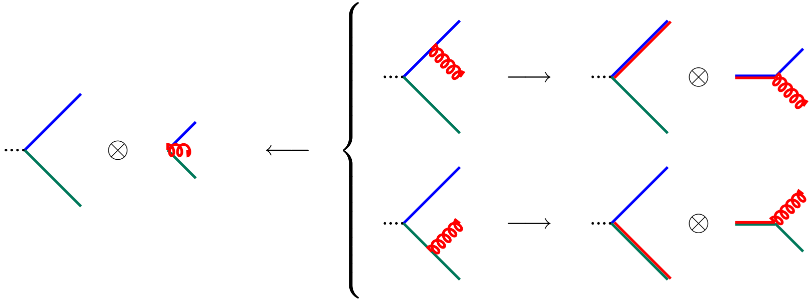
- $\mathcal{E}$ prescription [Frixione,Kunszt,Signer]
- Dipole formalism [Catani,Seymour]
- Antenna formalism [Kosower;Campbell,Cullen,Glover]
- More recent developments [Nagy, Somogyi, Trocsanyi;...]

We use antenna functions to construct subtraction terms

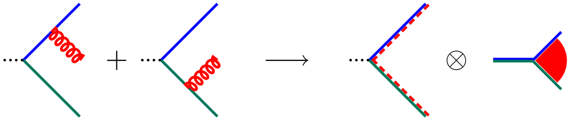
+ Less complicated than dipoles (1 Antenna $\simeq$ 2 Dipoles)
+ Extension to NNLO is feasible [Gehrmann-De Ridder,Gehrmann,Glover]
+ Antenna functions are constructed from physical matrix elements
+ Can be matched to parton shower [Giele,Kosower,Skands]

Soft and collinear factorization

Matrix elements factorize in the soft and collinear limit



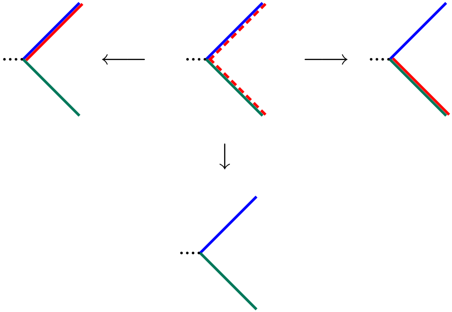
Antenna subtraction interpolates soft and collinear limits



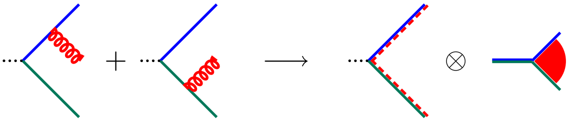
Phase-space Mapping

The phase-space mapping $1 + 3 + 2 \to 13 + 23$

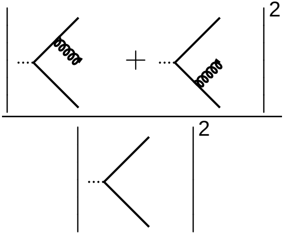
- Smoothly "distributes" the momentum of the radiation
- Momentum conservation
- Redefined momenta are on-shell
- Appropriate soft and collinear limits



Antenna Functions



$$A_{qg\bar{q}} =$$

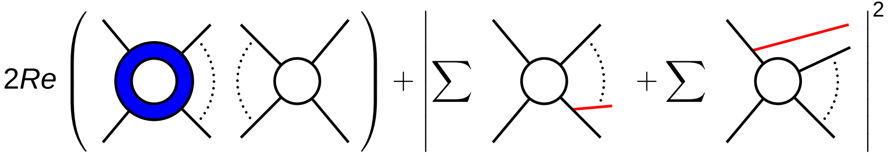


- Contains by construction soft and collinear emission between two color connected hard radiators.
- Are computed from ME of physical processes

Hadronic Reaction

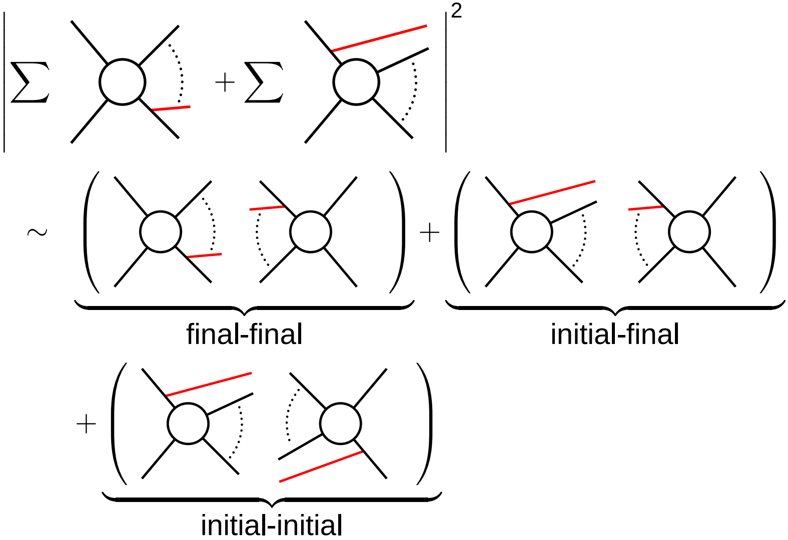
NLO corrections with hadronic initial state

$d\sigma^{\mathrm{NLO}} =$



- Final-state radiation $\rightarrow$ Virtual corrections
- Initial-state radiation $\rightarrow$ Virtual corrections+pdf factorization

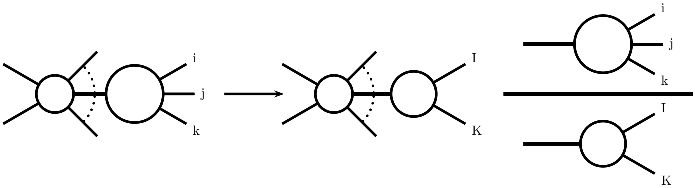
Hadronic Reaction



Final-final Subtraction

ME squared: $\left|\mathcal{M}_{m+1}(k_1,\ldots,k_i,k_j,k_k,\ldots,k_{m+1};p_1,p_2)\right|^2$

ME squared, subtraction:
$X_{ijk}(k_i,k_j,k_k)\,|\mathcal{M}_m(k_1,\ldots,K_I,K_K,\ldots,k_{m+1},p_1,p_2)|^2$



Phase space:
$d\Phi_{m+1}(k_1,\ldots,k_i,k_j,k_k,\ldots,k_{m+1};p_1,p_2)$

Factorized phase space:
$d\Phi_X(k_i,k_j,k_k)\times d\Phi_m(k_1,\ldots,K_I,K_K,\ldots,k_{m+1},p_1,p_2)$

Final-Final Subtraction term

Integrated antenna functions

$$\mathcal{X}_{ijk} = d\Phi_X(k_i, k_j, k_k) X_{ijk}(k_i, k_j, k_k)$$

$$\mathcal{X}_{ijk} \sim \frac{a}{\epsilon^2} + \frac{b}{\epsilon} + \mathcal{O}(\epsilon^0)$$

$$\mathcal{A}_{qg\bar{q}} = -2\mathbf{I}^{(1)}_{q\bar{q}} + \mathcal{O}(\epsilon^0)$$

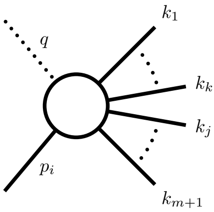
For combining with virtual corrections

$$d\sigma^{R,S} = \mathcal{N} \sum_{i,j,k} \mathcal{X}_{ijk} \int d\Phi_m(\ldots, K_I, K_K, \ldots) \left|\mathcal{M}_m(\ldots, K_I, K_K, \ldots)\right|^2$$

For combining with real corrections

$$d\sigma^{R,S} = \mathcal{N} \int d\Phi_{m+1}(\ldots k_i, k_j, k_l \ldots) \sum_{i,j,k} X_{ijk}(k_i, k_j, k_k) \left|\mathcal{M}_m(\ldots K_I, K_K \ldots)\right|^2$$

Initial-final Phase-space Mapping



One more condition $P = xp$

$$\left.\begin{matrix} p \\ k_j \\ k_k \end{matrix}\right\} \to \left\{\begin{matrix} P = xp \\ K_K \end{matrix}\right.$$

$$x = \frac{s_{ij} + s_{ik} - s_{jk}}{s_{ij} + s_{ik}}$$

$$K_K = k_j + k_k - xp$$

Limits

$$xp \to p, \quad K_K \to k_k, \quad \text{when } j \text{ becomes soft}$$

$$xp \to p, \quad K_K \to k_j + k_k, \quad \text{when } k_j \text{ collinear with } k_k$$

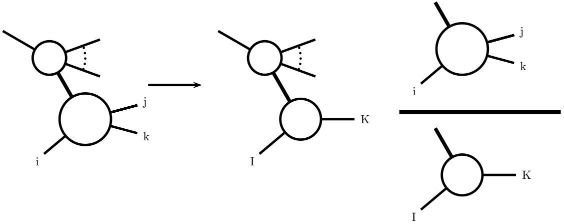
$$xp \to p - k_j, \quad K_K \to k_k, \quad \text{when } k_j \text{ collinear with } p$$

Initial-final Subtraction

ME squared: $\left|\mathcal{M}(k_1,\ldots,k_j,k_k,\ldots,k_{m+1};q,p)\right|^2$

ME squared, subtraction:

$X_{ijk}(k_j,k_k;p)\left|\mathcal{M}(k_1,\ldots,K_K,\ldots,k_{m+1},q,xp)\right|^2$



Phase space: $d\Phi_{m+1}(k_1,\ldots,k_j,k_k,\ldots,k_{m+1};q,p)$

Factorized phase space:

$\int \frac{dx}{x} d\Phi_X(k_j,k_k;q,p)\mathcal{J}(k_j,k_k,p)d\Phi_m(k_1,\ldots,K_K,\ldots,k_{m+1},q,xp)$

Subtraction Term for Initial-Final

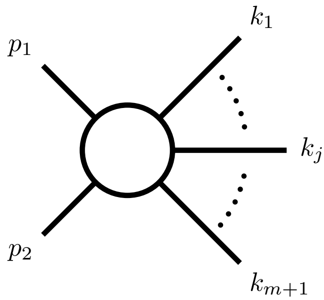
Integrated antenna functions

$$\mathcal{X}_{i,jk}(x) = d\Phi_X(k_j,k_k;p,q)X_{i,jk}(k_j,k_k;p)\mathcal{J}(k_j,k_k;p)$$

$$\mathcal{A}_{q,gq}(x) = -2\mathbf{I}^{(1)}_{q\bar{q}}\,\delta(1-x) + (Q^2)^{-\epsilon}\left[-\frac{1}{2\epsilon}P_{qq}(x) + \mathcal{O}(\epsilon^0)\right]$$

$$\begin{aligned} d\sigma^{NLO,S}_{ijk} \\ &= \mathcal{N}\sum_{ijk}\mathcal{X}_{i,jk}(x)d\Phi_m(k_1,\ldots,K_K,\ldots,k_{m+1};q,xp) \\ &\quad\times|\mathcal{M}_m(k_1,\ldots,K_K,\ldots,k_{m+1};xp)|^2 \end{aligned}$$

Initial-initial Phase-space Mapping



- Need both $P_1 = x_1 p_1$ and $P_2 = x_2 p_2$
- Apply boost on all spectator particles

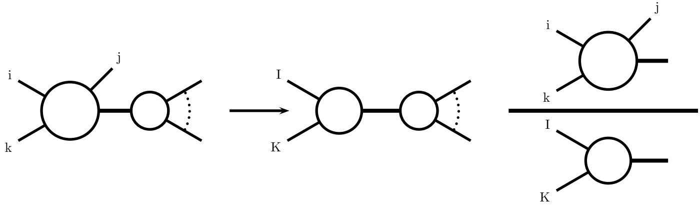
$$K \equiv p_1 + p_2 - p_j = k_1 + ... + k_{j-1} + k_{j+1} + ... + k_{m+1}$$

$$\rightarrow \tilde{K} \equiv x_1 p_1 + x_2 p_2 = \tilde{k}_1 + ... + \tilde{k}_{j-1} + \tilde{k}_{j+1} + ... + \tilde{k}_{m+1}$$

- Boost vanishes for soft or collinear k_j

ME squared: $\left|\mathcal{M}_{m+1}(k_1,\ldots,k_{j-1},k_j,k_{j+1},\ldots,k_{m+1};p_i,p_j)\right|^2$

Phase space: $d\Phi_{m+1}(k_1,\ldots,k_{j-1},k_j,k_{j+1},\ldots,k_{m+1};p_1,p_2)$



ME squared, subtraction:

$$X_{12,j}(k_j;p_1,p_2)\left|\mathcal{M}_m(\tilde{k}_1,\ldots,\tilde{k}_{j-1},\tilde{k}_{j+1},\ldots,\tilde{k}_{m+1};x_1p_1,x_2p_2)\right|^2$$

Phase Space Factorization

Phase space:

$$d\Phi_{m+1} = \int dx_1 dx_2 d\Phi_X(k_j; p_1, p_2)\delta(x_1 - \hat{x}_1)\delta(x_2 - \hat{x}_2) \times \mathcal{J}(k_j, p_1, p_2) d\Phi_m(\tilde{k}_1, ..., \tilde{k}_{j-1}, \tilde{k}_{j+1}, ..., \tilde{k}_{m+1}; x_1 p_1, x_2 p_2)$$

with

$$\hat{x}_1 = \sqrt{\frac{s_{12} - s_{j2}}{s_{12}} \frac{s_{12} - s_{1j} - s_{j2}}{s_{12} - s_{1j}}},$$

$$\hat{x}_2 = \sqrt{\frac{s_{12} - s_{1j}}{s_{12}} \frac{s_{12} - s_{1j} - s_{j2}}{s_{12} - s_{j2}}}.$$

Integrated Antenna Function

Integrated Antenna function

$$\mathcal{X}_{i,jk}(x_1,x_2) = \int d^d k_j \delta(k_j^2)\delta(x_1-\hat{x}_1)\delta(x_2-\hat{x}_2)X_{12,j}(k_j;p_1,p_2)\mathcal{J}(k_j;p_1,p_2)$$

Phase space integration freezes

$$\begin{aligned}\mathcal{A}_{q\bar{q},g} &= -2\delta(1-x_1)\delta(1-x_2)\mathbf{I}^{(1)}_{q\bar{q},g} \\ &\quad -s^{-\epsilon}\frac{P_{qq}(x_1)\delta(1-x_2)+P_{qq}(x_2)\delta(1-x_1)}{2\epsilon}+\mathcal{O}(\epsilon)\end{aligned}$$

Final Form of the Subtraction Term

$$
\begin{aligned}
& d\sigma_{NLO}^{S,(ii)} \\
&= \mathcal{N} d\Phi_{m+1}(k_1,\ldots,k_{j-1},k_j,k_{j+1},\ldots,k_{m+1};p_1,p_2) \\
&\quad \times \sum_j X_{12,j}^0 \left| \mathcal{M}_m(\tilde{k}_1,\ldots,\tilde{k}_{j-1},\tilde{k}_{j+1},\ldots,\tilde{k}_{m+1};x_1 p_1,x_2 p_2) \right|^2 \\
&= \mathcal{N} \sum_j \mathcal{X}_{12,j}(x_1,x_2) d\Phi_m(\tilde{j}_1,\ldots,\tilde{k}_{j-1},\tilde{k}_{j+1},\tilde{k}_{m+1};x_1 p_1,x_2 p_2) \\
&\quad \times \left| \mathcal{M}_m(\tilde{k}_1,\ldots,\tilde{k}_{j-1},\tilde{k}_{j+1},\ldots,\tilde{k}_{m+1};x_1 p_1,x_2 p_2) \right|^2
\end{aligned}
$$

Conclusions

- Antenna subtraction for processes with initial state partons (at NLO)
- Method can be extended to NNLO (work in progress)
- Possible applications
  - Matching to antenna-based parton shower
  - NNLO $t\bar{t}$ production
  - NNLO $pp \to 2j$ production
  - NNLO DIS $(2+1)j$ production