

# **NLO QCD Corrections to Weak Boson Pair Production via Vector Boson Fusion**



**Barbara Jäger**

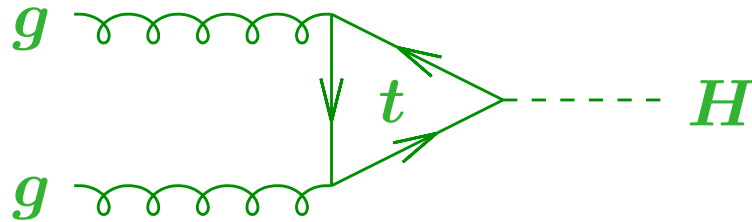
**KEK Theory Group**

# Outline

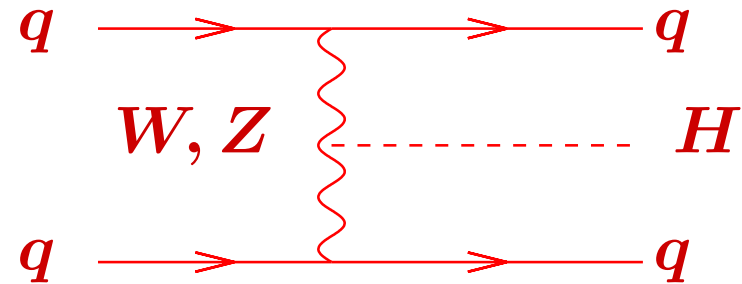
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- ✗ motivation: why vector boson fusion?
- ✗ focus on  $VV$  production in VBF
  - ☞ theoretical concepts & techniques
  - ☞ results
- ✗ summary & conclusions

# Higgs production at hadron colliders



$gg$  fusion

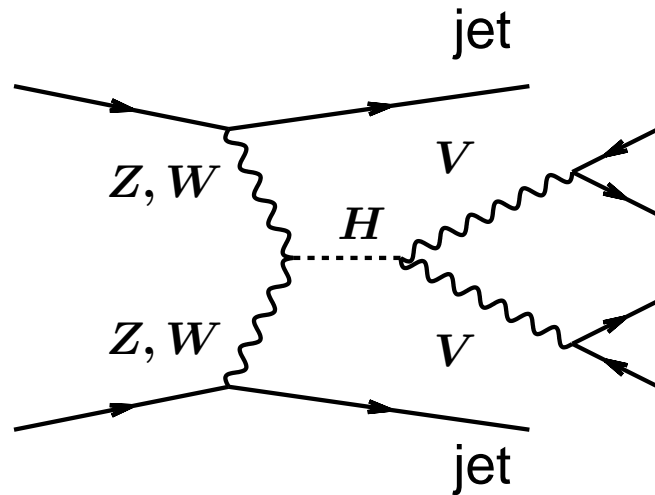


vector boson fusion (VBF)

expected statistical & systematic errors on  $\sigma \cdot B$ :

- luminosity / acceptance uncertainties:  $\pm 5\%$
- QCD / PDF uncertainties:  
 $\pm 15\%$  for gluon fusion                       $\pm 5\%$  for VBF

# Higgs production in VBF

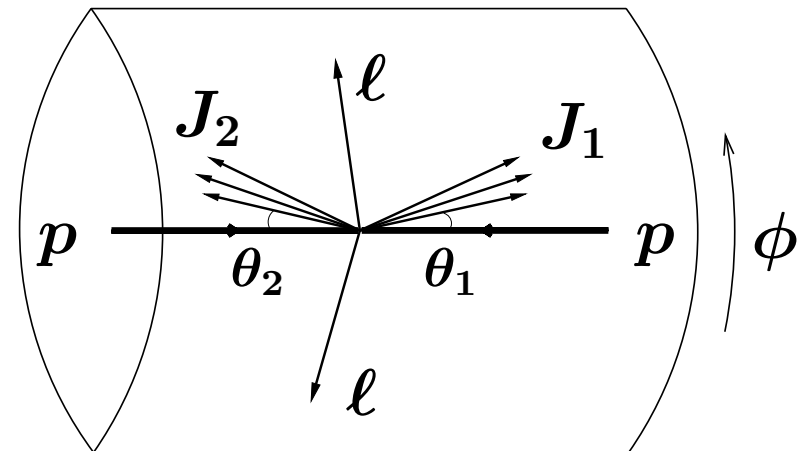


scattered quarks

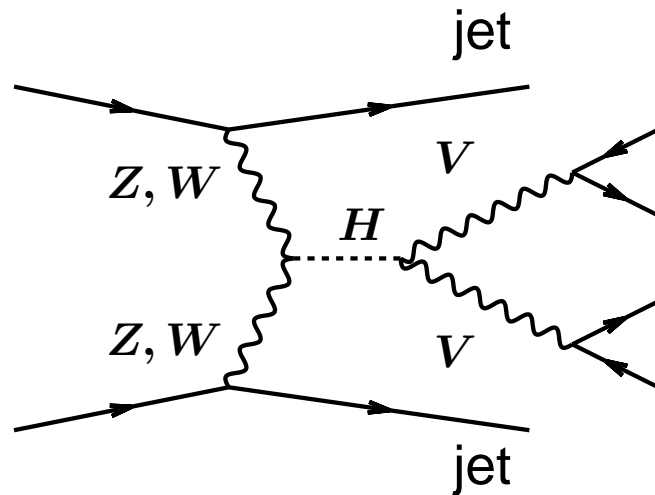
→ two forward tagging jets  
(energetic;  $p_T > 20$  GeV)

Higgs decay products  
typically between tagging jets

little jet activity in  
central rapidity region  
(colorless  $V$  exchange  
→ gluon radiation suppressed )



# Higgs production in VBF



inclusive cross section:

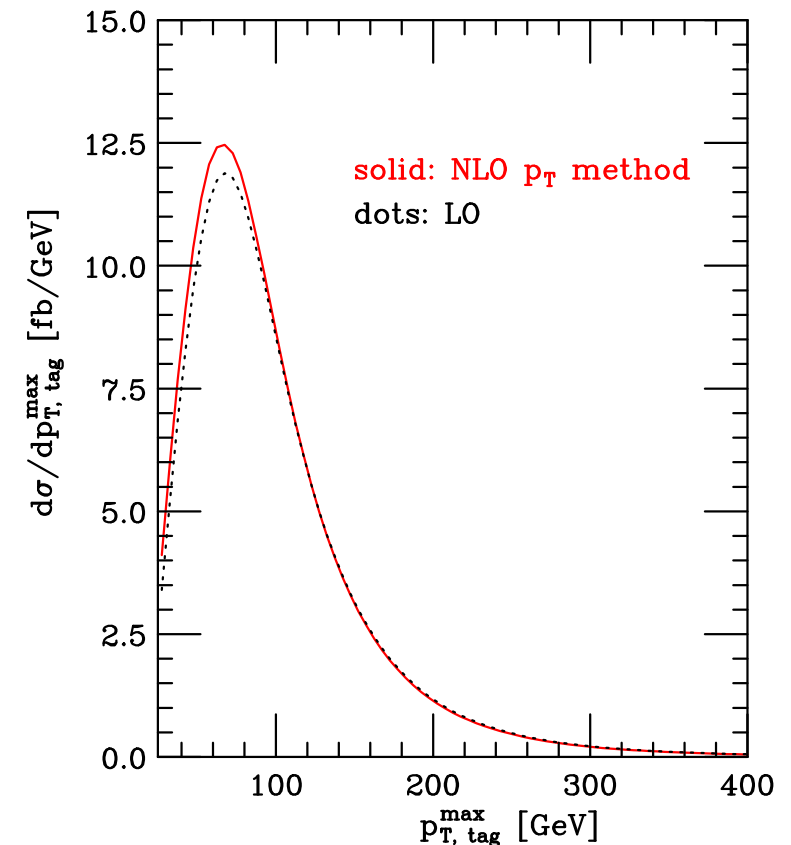
*Han, Valencia, Willenbrock (1992)*

distributions:

*Figy, Oleari, Zeppenfeld (2003)*

*Berger, Campbell (2004)*

NLO QCD corrections  
moderate and  
theoretically well under control



# Needed:

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establishing a signal for

$$H \rightarrow W^+ W^- \rightarrow e^\pm \mu^\mp \cancel{p}_T$$

$$H \rightarrow ZZ \rightarrow e^+ e^- \mu^+ \mu^-$$

$$H \rightarrow ZZ \rightarrow e^+ e^- \cancel{p}_T$$

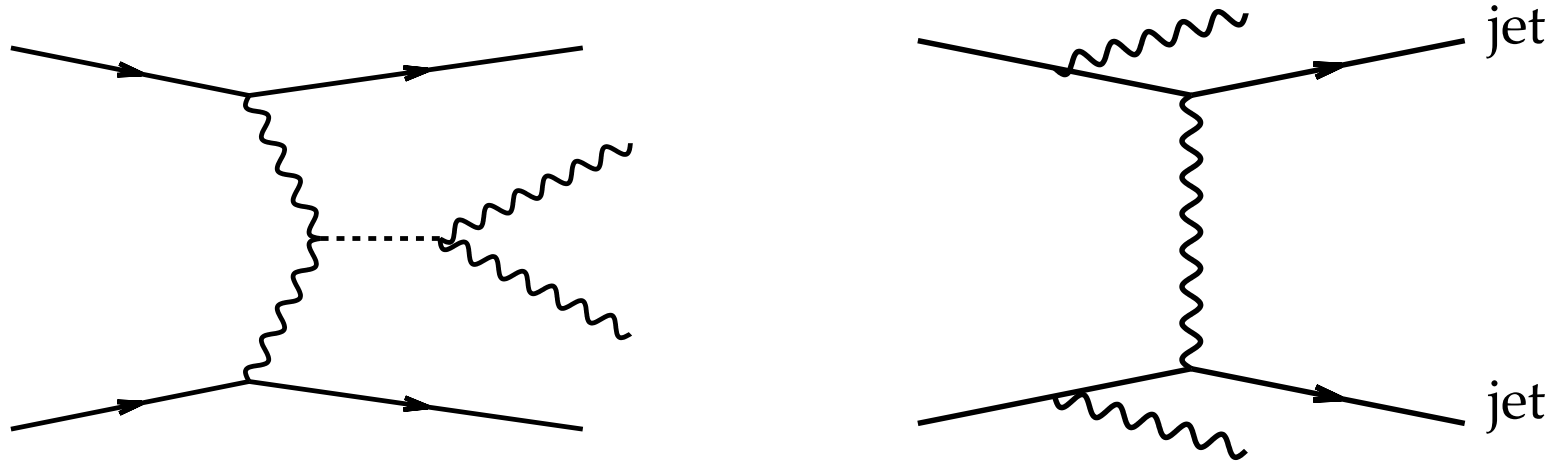
in VBF requires:

- calculation of **signal and background distributions**  
(ideally: NLO QCD predictions to match  
statistical accuracy of LHC)
- identification of **suitable cuts** to isolate the signal

*Rainwater, Zeppenfeld (1999)*

*Kauer, Plehn, Rainwater, Zeppenfeld (2000)*

# an irreducible background



$pp \rightarrow VV + jj$  via VBF

(EW  $VV + jj$  production):

similar characteristics to  $H$  signal process

→ background rejection difficult

# VV scattering & VBF

$qq \rightarrow qqVV$  via VBF

- info on **gauge boson interactions**:  
(quartic) EW gauge boson vertex  
anomalous gauge boson couplings

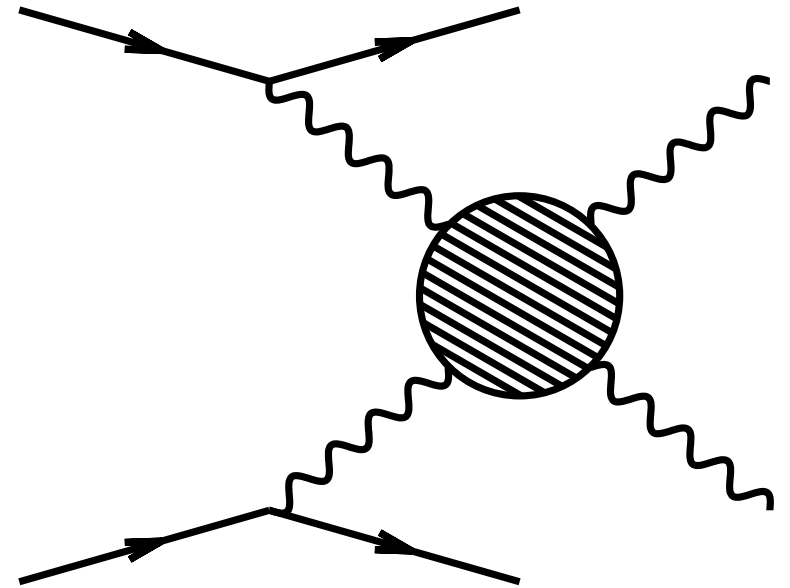
- gives access to  $V_L V_L \rightarrow V_L V_L$



**electroweak symmetry breaking:**

light Higgs boson?  $M_H$ ?

strong EWSB? specific mechanism?



# Weak boson scattering in the literature

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- ✘  $qq \rightarrow qqVV$  within the “Effective  $W$  Approximation”:  
*Cahn, Dawson (1984); Dawson (1985);*  
*Duncan et al. (1986); Dobado et al. (1991)*
- ✘ full  $qq \rightarrow qqVV$  (without  $W$  decay /  $W$  decay in NWA):  
*Gunion et al. (1986); Dicus, Vega (1986);*  
*Baur, Glover (1990); Barger et al. (1991f);*
- ✘ application to strongly interacting gauge boson systems:  
*e.g. Bagger et al. (1994f)*
- ✘ LO event generator for six-fermion processes at the LHC  
*Accomando et al. (2005f)*
- ✘ Study of the quartic electroweak gauge boson couplings  
*Eboli et al. (2006)*
- ✘ ...

# EW $VVjj$ production

need **stable, fast & flexible Monte Carlo program** allowing for

- computation of various jet observables  
at NLO-QCD accuracy
- straightforward implementation of cuts

[ C. Oleari, D. Zeppenfeld, B. J., 2006 ]

[ G. Bozzi, C. Oleari, D. Zeppenfeld, B. J., 2007 ]

major challenges:

- **multi-parton** process:  $2 \rightarrow 4$  for  $qq \rightarrow qq VV$ ;  
 $2 \rightarrow 6$  for  $qq \rightarrow qq \ell^+ \ell^- \nu_e \bar{\nu}_e$   
or  $qq \rightarrow qq \ell^+ \ell^- \ell^+ \ell^-$
- full consideration of **finite width effects**
- numerically stable treatment of **pentagon** contributions

# $pp \rightarrow \ell\bar{\ell} \ell'\bar{\ell}' jj$ : the leading order

need to compute numerical value for

$$|\mathcal{M}_B|^2 = \left| \begin{array}{c} \text{diagram 1} \\ + \\ \text{diagram 2} \\ + \\ \text{diagram 3} \\ + \dots \end{array} \right|^2$$

at each generated phase space point in 4 dim (finite)

... depending on leptonic final state: up to 580 diagrams

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essential: organize calculation economically

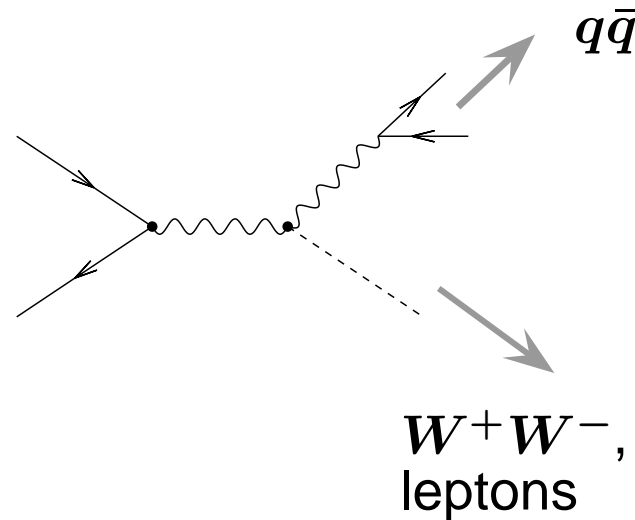


- develop **modular structure**
- compute each building block only once per phase space point

# Not Included

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- interference effects from diagrams obtained by interchanging **identical** initial- or final-state (anti)**quarks**
- identical flavor **annihilation processes** with subsequent decay into quarks and similar contributions like



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- identical flavor **annihilation processes** with subsequent decay into quarks and similar contributions

neglected terms strongly suppressed in PS region  
where VBF can be observed experimentally

(require two widely separated  
quark jets of large invariant mass)

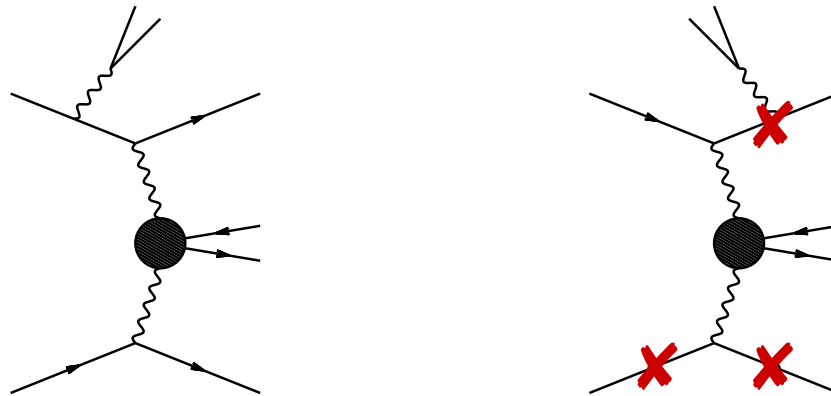
. . . both contributions on sub-percent level  
for related VBF processes (checked)

*see: Oleari, Zeppenfeld, hep-ph/0310156*

*Georg, diploma thesis, Karlsruhe (2005)*

# Practical Implementation

- compute polarization vectors, quark currents etc. by hand in **helicity amplitude formalism** of *Hagiwara, Zeppenfeld (1986)*
  - combine them with **leptonic tensors** adapted from MadGraph generated code
  - **recycle** all building blocks emerging repeatedly (related sub-diagrams, various flavor combinations, crossed processes ...)



such recycling is used to a very small extent by MadGraph/MadEvent (within each sub-process and esp. for different sub-processes)

# Efficiency

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✘ employ **MadGraph** for computing first reference result  
but: repeated calculation of similar diagrams makes code  
extremely **slow**

(1 ÷ 2 months CPU time on a 3 GHz Linux PC for  $\Delta\sigma/\sigma \approx 0.2\%$   
for  $WW$  and even more in  $ZZ$ -case)

✘ high statistics needed especially for kinematic distributions

✘ **pre-calculate leptonic tensors** (for full NLO program)

☞ **gain speed-up of factor 70** for full code

✘ valuable check: comparison of `vbfnlo` to  
result obtained with `MadGraph`

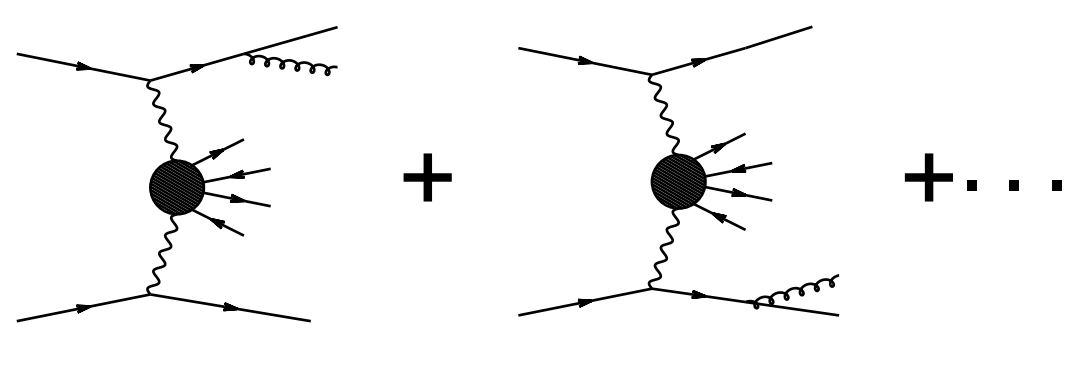
# Outline of the NLO Calculation

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- ✘ calculation of  $d\hat{\sigma}$  at  $\mathcal{O}(\alpha^6\alpha_s)$  (NLO QCD)
  - dimensional reduction ( $d = 4 - 2\epsilon$ )
  - $\overline{\text{MS}}$ -renormalization
- ✘ handling of **infrared singularities** by dipole subtraction approach of *Catani & Seymour*
- ✘ need to compute
  - real emission contributions
  - counterterms
  - virtual corrections
- ✘ **phase space integration** and convolution with **PDFs** with Monte Carlo techniques in 4 dimensions

# Real Emission Contributions

needed: numerical value for up to 2892 diagrams ( $ZZjj$  case)

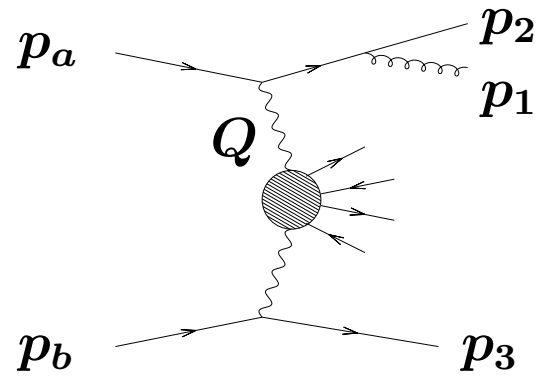
$$|\mathcal{M}_R|^2 = \left| \begin{array}{c} \text{Diagram 1} \\ + \\ \text{Diagram 2} \\ + \dots \end{array} \right|^2$$
The equation shows the squared magnitude of the real emission amplitude,  $|\mathcal{M}_R|^2$ , as the square of a sum of diagrams. The first diagram shows a central black circle with four incoming lines (two solid, two wavy) and four outgoing lines (two solid, two wavy). The second diagram is similar but with the wavy lines swapped. Ellipses indicate more diagrams. A large vertical bar encloses the sum, with a '2' as a superscript on the right.

at each generated phase space point **in 4 dimensions**  
→ apply same techniques as at LO

- ☹ the major challenge: large number of diagrams  
(without optimization **code extremely slow!**)
- ☺ the solution: apply **speed-up tricks** developed at LO  
(here even more effective)

still: MadGraph extensively used for debugging and cross checks

# Counterterms



upper & lower quark lines “decoupled”

→ simple singularity structure  
with counterterms

$$\frac{8\pi\alpha_s(\mu_r)}{Q^2} C_F \frac{x^2 + z^2}{(1-x)(1-z)} |\mathcal{M}_B(\tilde{p})|^2$$

... continuous interpolation between **soft and collinear**  
gluon radiation ( $x$  and / or  $z \rightarrow 1$ )

analytical integration over gluon phase space gives

$$|\mathcal{M}_B(p)|^2 F(Q) \left[ \frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \text{const.} \right]$$

# Virtual Corrections

... interference of LO diagrams with

$$\mathcal{M}_V = \text{[diagram 1]} + \text{[diagram 2]} + \text{[diagram 3]} + \dots$$
$$= \mathcal{M}_B F(Q) \left[ -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} \right] + \tilde{\mathcal{M}}_V^{finite}$$

$\tilde{\mathcal{M}}_V^{finite}$  computed with Passarino-Veltman reduction  
cumbersome: (numerically small) **pentagon** contributions

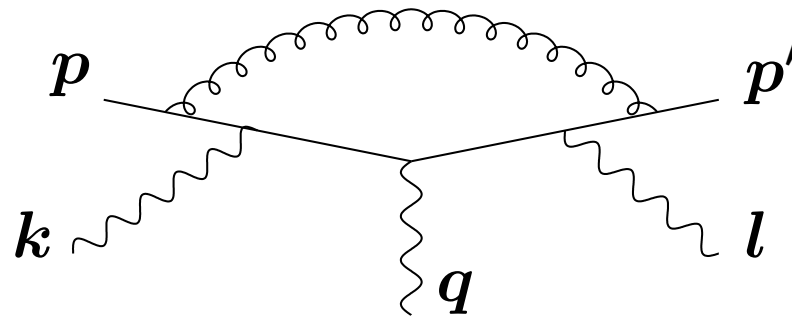
**combination** of real emission and virtual contributions with  
subtraction terms according to dipole approach of  
*Catani & Seymour*

poles canceled analytically  $\rightarrow$  **finite** results

# Pentagon Contributions

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$$\mathcal{M}_5 = \varepsilon_\mu(k)\varepsilon_\nu(l)j_\rho(q)P^{\mu\nu\rho}(p, k, q, l)$$



planar configurations with linearly dependent momenta  
→ trouble with Passarino-Veltman reduction

but: **singularity unphysical!**

→ perform **interpolation** to “safe” regions of phase space

# Pentagon Contributions

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further improvement by **gauge invariant decomposition**:

$$\varepsilon_\mu(k) \rightarrow \varepsilon'_\mu(k) = \varepsilon_\mu(k) - \beta k_\mu$$

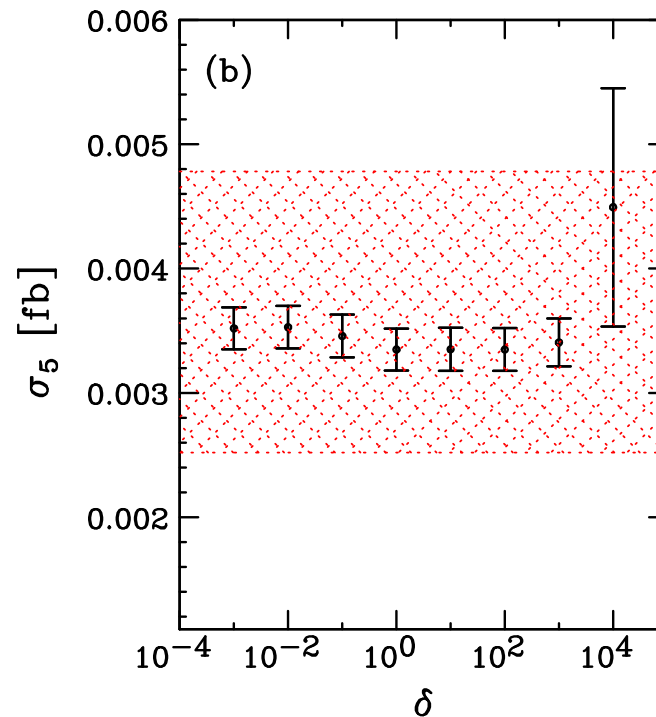
$$\text{use } k_\mu \mathcal{E}^{\mu\nu\rho}(p, k, q, l) = \mathcal{D}^{\nu\rho}(p, k + q, l)$$

$$\begin{aligned} \mathcal{M}_5 &= \left[ \varepsilon'_\mu(k) + \beta k_\mu \right] \varepsilon_\nu(l) j_\rho(q) \mathcal{E}^{\mu\nu\rho}(p, k, q, l) \\ &= \varepsilon'_\mu(k) \varepsilon_\nu(l) j_\rho(q) \mathcal{E}^{\mu\nu\rho}(p, k, q, l) \\ &\quad + \beta \varepsilon_\nu(l) j_\rho(q) \mathcal{D}^{\nu\rho}(p, k + q, l) \end{aligned}$$

proper choice of  $\beta \rightarrow$  remaining “true” pentagon small  
box-type contributions numerically stable

# Pentagon Contributions

numerical stability of genuine pentagon contributions:  
check Ward identities for each phase space point and  
keep only satisfactory events (violation  $\delta \lesssim 10\%$ )



even better: apply new reduction method that **avoids**  
**inverse Gram determinants** [Denner, Dittmaier (2002, 2005)]

# Checks

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- ✓ comparison of LO and real emission  
amplitudes with MadGraph

- ✓ soft / collinear limits:  $d\sigma^R \rightarrow d\sigma^A$

- ✓ QCD gauge invariance of real emission contributions:

$$\mathcal{M} = \varepsilon_{\mu}^{\star}(p_g) \mathcal{M}^{\mu} = \left[ \varepsilon_{\mu}^{\star}(p_g) + C p_{g\mu} \right] \mathcal{M}^{\mu}$$

- ✓ EW gauge invariance of virtual contributions

- ✓ produce independent code for NC amplitudes

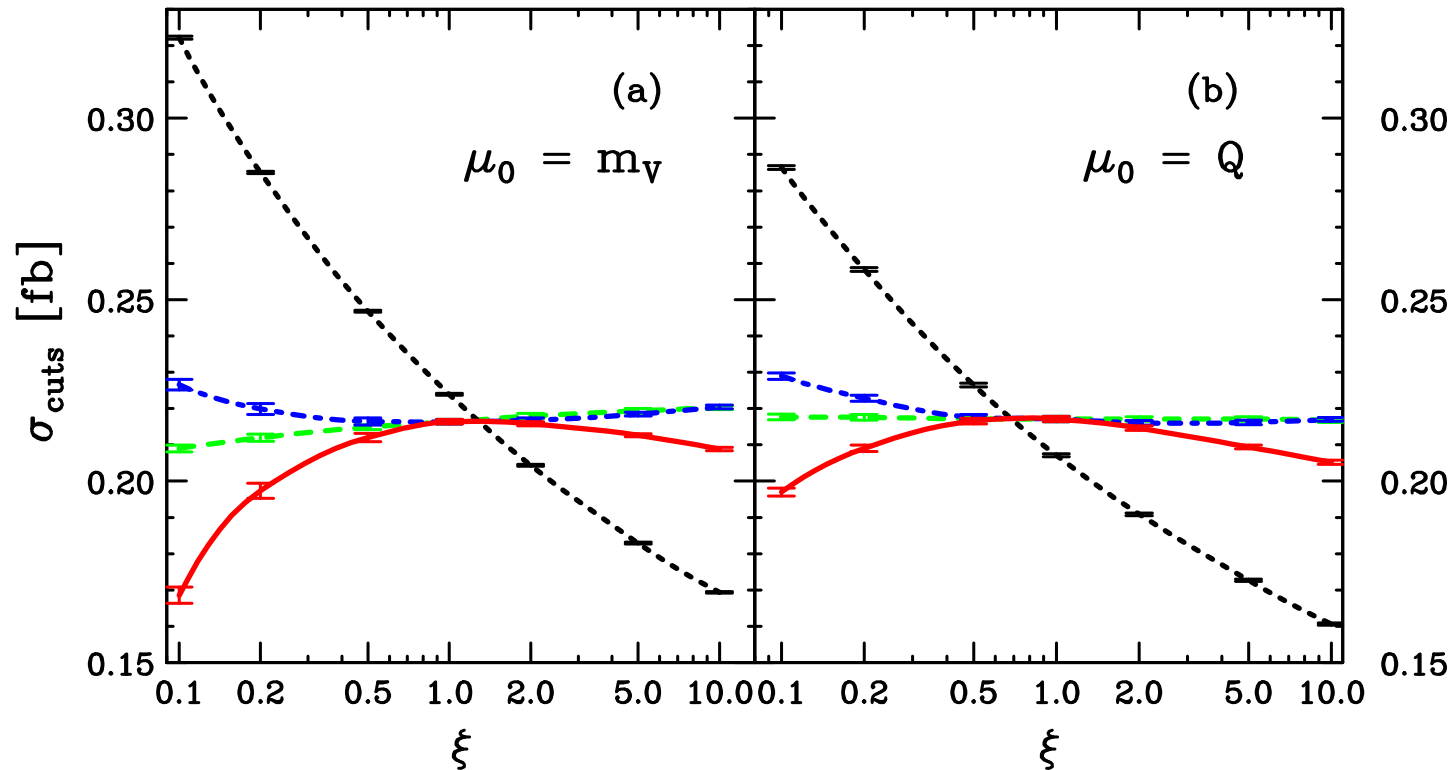
- ✓ comparison of LO result to MadEvent (generic cuts)

# Results: $pp \rightarrow VV jj$ @ LHC

using  $k_T$  algorithm, CTEQ6 parton distributions,  
and applying following cuts:

tagging jets	$p_{Tj} \geq 20 \text{ GeV},  y_j  \leq 4.5,$ $\Delta y_{jj} =  y_{j_1} - y_{j_2}  > 4,$ $(M_{jj} > 600 \text{ GeV})$ <p>jets located in opposite hemispheres</p>
charged leptons	$p_{T\ell} \geq 20 \text{ GeV},  \eta_\ell  \leq 2.5, \Delta R_{j\ell} \geq 0.4,$ $y_{j,\min} < \eta_\ell < y_{j,\max}$
	$M_H = 120 \text{ GeV}, M_{WW,ZZ} > 130 \text{ GeV}$ <p>(for <math>WW</math> and <math>ZZ</math> case: <math>VV</math> continuum only)</p>

# Scale Uncertainty: $pp \rightarrow W^+ Z jj$



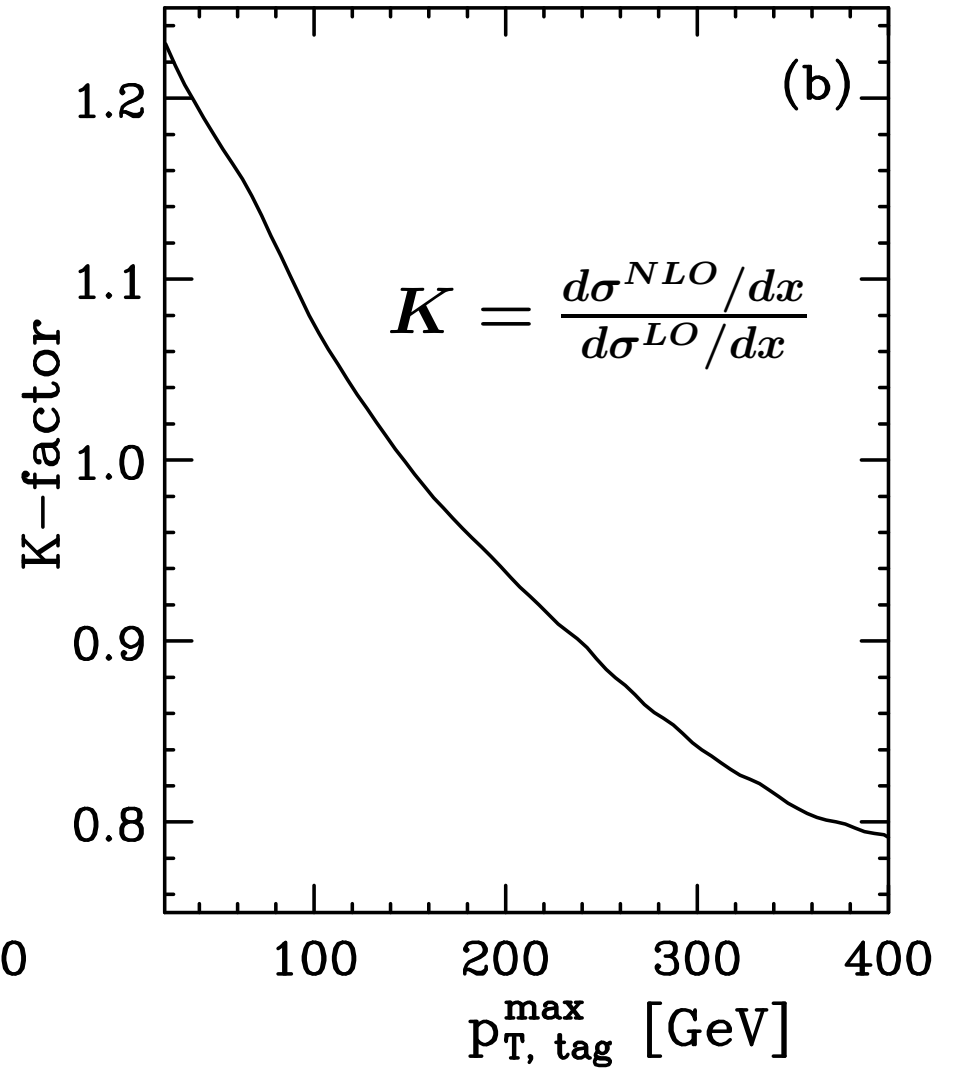
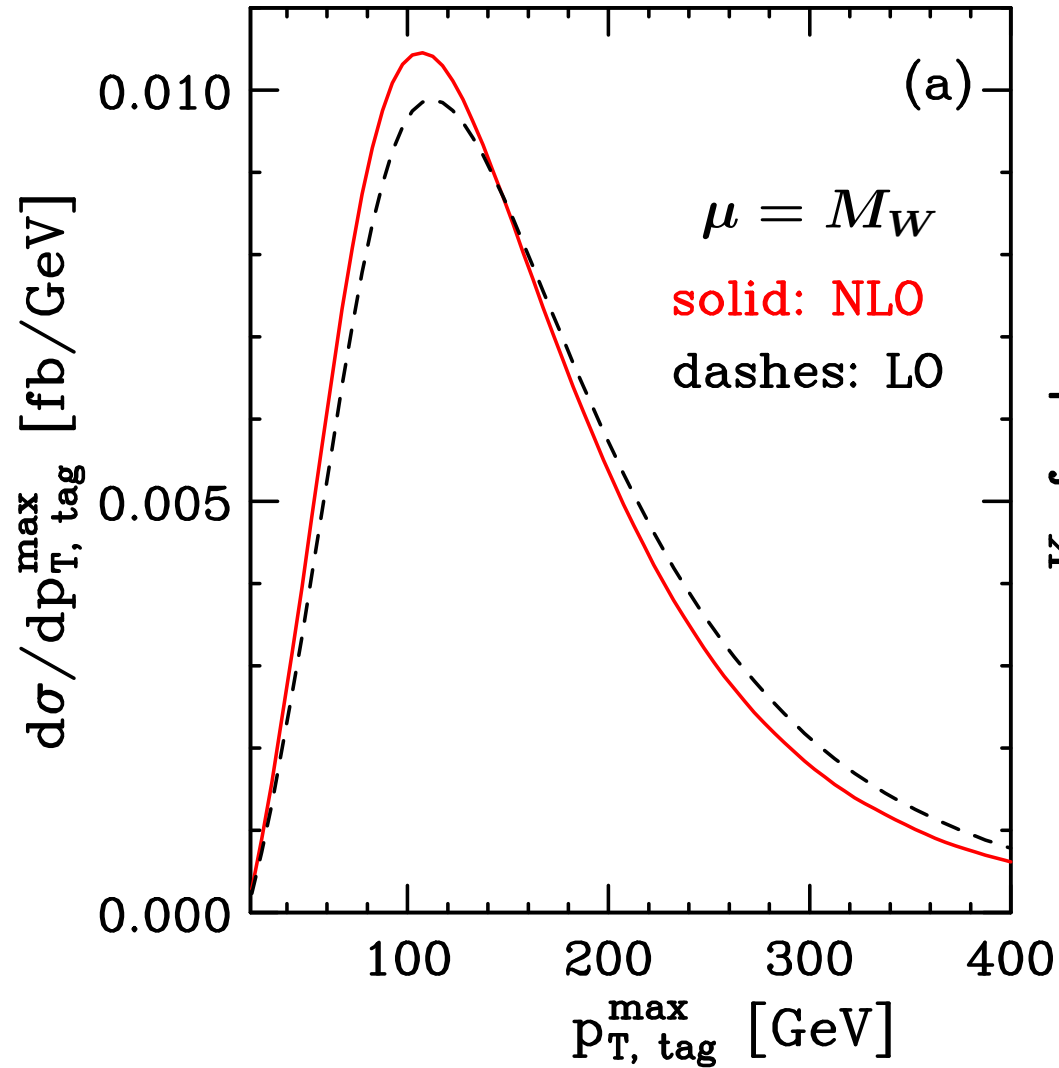
choose  $\mu_R = \xi\mu_0$  and  $\mu_F = \xi\mu_0$  with variable  $\xi$   
and set  $\mu_0 = m_V$  or  $\mu_0 = Q$



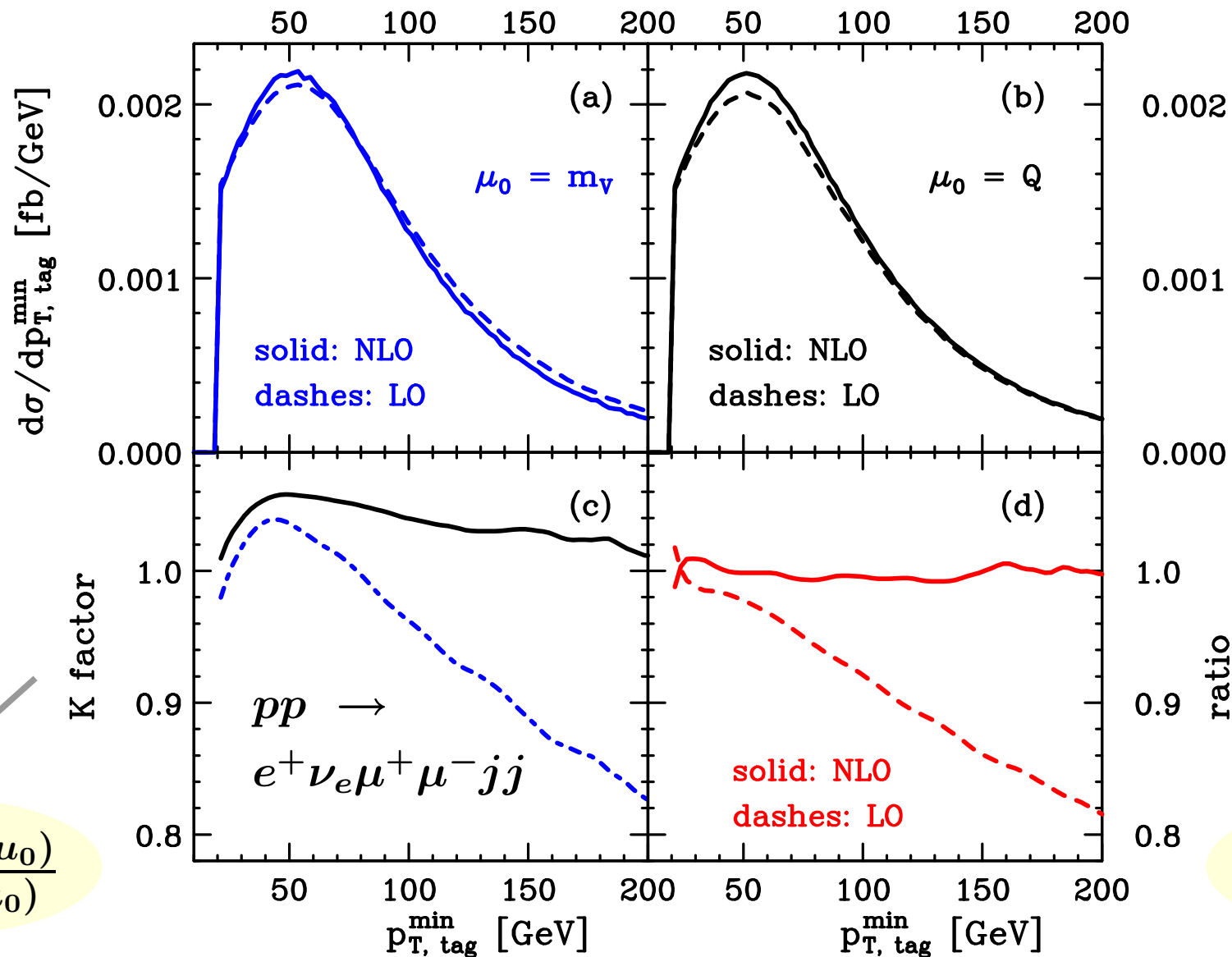
LO: no control on scale

NLO: scale dependence strongly reduced

# Distributions: $pp \rightarrow W^+W^-jj$



# Scales matter (at LO) ...



$$\frac{\text{Obs}^{NLO}(\mu_0)}{\text{Obs}^{LO}(\mu_0)}$$

$$\frac{\text{Obs}(\mu_0=m_V)}{\text{Obs}(\mu_0=Q)}$$

# Angular Distribution of Charged Leptons

in  $H \rightarrow W^+W^-$ : spins anti-correlated



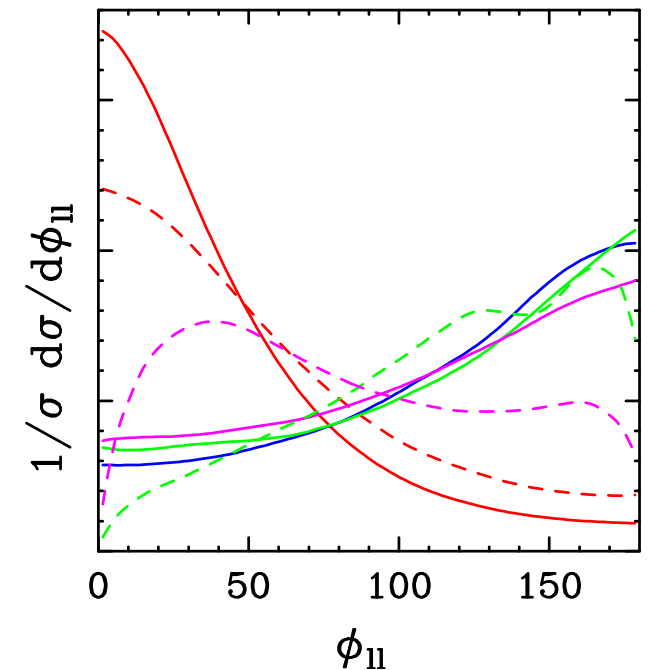
leptons emitted preferentially in same direction

no such correlation, if  $W$  bosons do not stem from the Higgs

*Dittmar, Dreiner (1996)*

distribution for EW  $W^+W^-$  production significantly different from Higgs signal

*Rainwater, Zeppenfeld (1999)*



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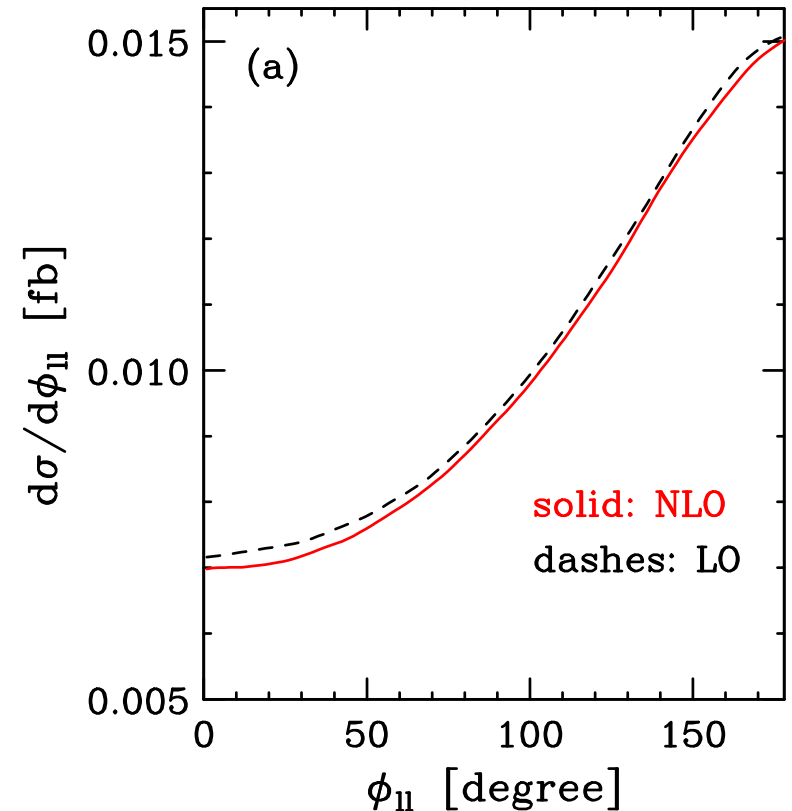


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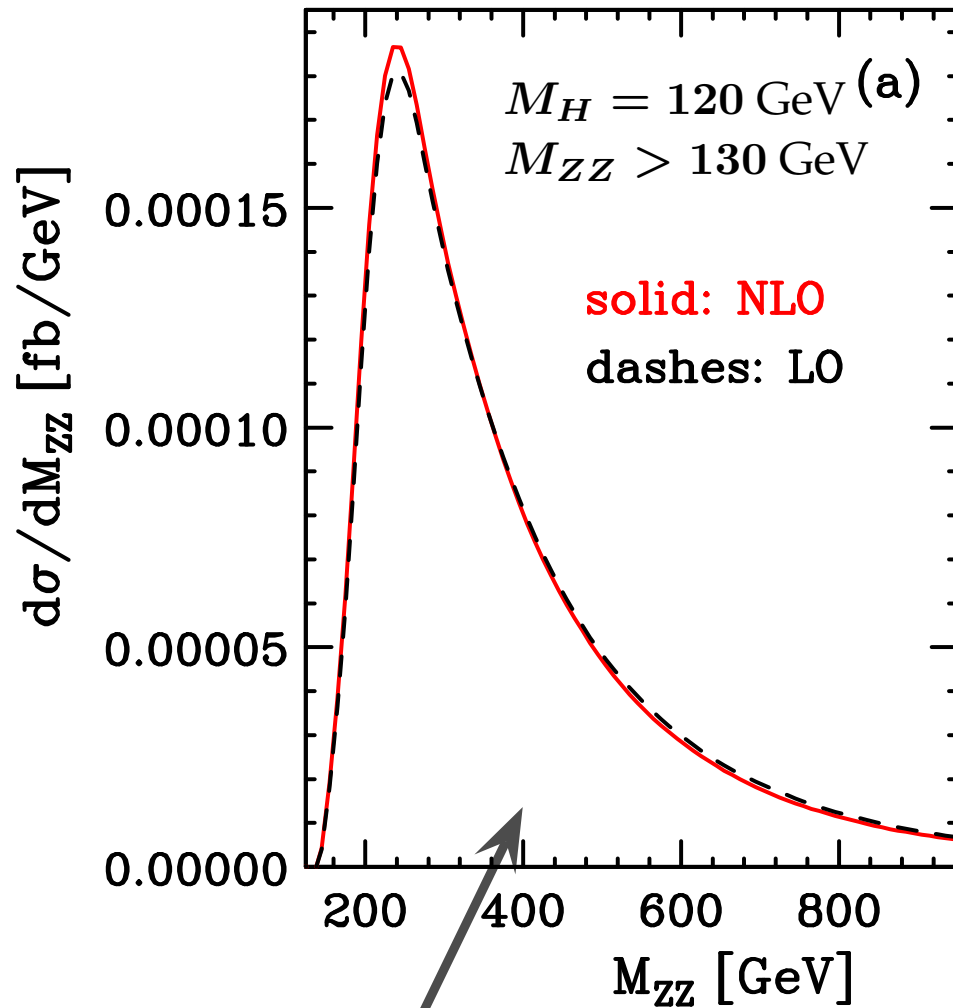
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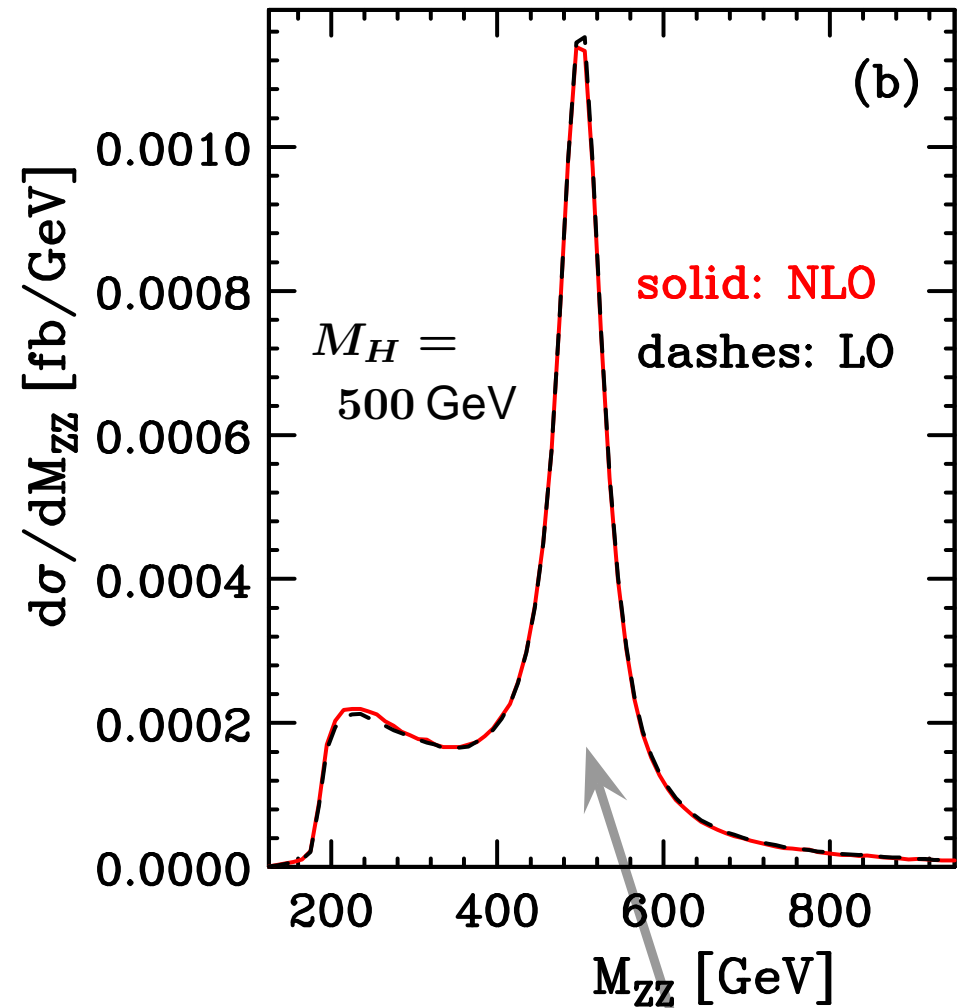


# Distributions: $pp \rightarrow \ell^+ \ell^- \ell'^+ \ell'^- jj$



background

VV production via  $\nu$ BF



background + signal

# Conclusions

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- ✘ VBF offers promising prospects for discovery of Higgs boson (or SEWSB ?) and investigation of its properties



need to know signal and background precisely

- ✘ developed fully flexible parton-level Monte Carlo program with NLO QCD cross sections and distributions for

$$pp \rightarrow W^+W^- jj \quad \text{and} \quad pp \rightarrow ZZ jj$$

$$pp \rightarrow W^+Z jj \quad \text{and} \quad pp \rightarrow W^-Z jj$$

(including leptonic decays)

- ✘ perturbative corrections under excellent control (modest scale uncertainties &  $K$ -factors)

- ✘ in progress: extension to related processes  $pp \rightarrow W^\pm W^\pm jj$

process ( $V \in \{Z, W, \gamma\}$ )	relevant for
1. $pp \rightarrow V V + \text{jet}$	$t\bar{t}H$ , new physics
2. $pp \rightarrow H + 2 \text{ jets}$	$H$ production by vector boson fusion (VBF)
3. $pp \rightarrow t\bar{t} b\bar{b}$	$t\bar{t}H$
4. $pp \rightarrow t\bar{t} + 2 \text{ jets}$	$t\bar{t}H$
5. $pp \rightarrow V V b\bar{b}$	VBF $\rightarrow H \rightarrow V V$ , $t\bar{t}H$ , new physics
6. $pp \rightarrow V V + 2 \text{ jets}$	VBF $\rightarrow H \rightarrow V V$
7. $pp \rightarrow V + 3 \text{ jets}$	various new physics signatures
8. $pp \rightarrow V V V$	SUSY trilepton searches

“The wishlist of processes for which a NLO calculation is both desired and feasible  
in the near future.”

# The 2006/07 Checklist

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have a look into the  
2006/07 gift box and discover

$$pp \rightarrow W^+ W^- jj \checkmark \quad pp \rightarrow ZZjj \checkmark$$

$$pp \rightarrow W^+ Zjj \checkmark \quad pp \rightarrow W^- Zjj \checkmark$$

at order  $\alpha^6 \alpha_s$

still on the wishlist:  $pp \rightarrow VVjj$  at order  $\alpha^4 \alpha_s^3$