Radiative corrections to co-annihilation processes

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1. Introduction
2. Analysis of stop co-annihilation
3. QCD corrections
4. Results

New physics models for dark matter

- Dark matter constitutes of

  Weakly **I**nteracting **M**assive **P**articles,

  that are stable due to new symmetry
  (SUSY, UED, Little Higgs, ...)

- Within model, relic density can be computed Gondolo, Gelmini '91

  Boltzmann equation $\frac{\mathrm{d}n}{\mathrm{d}t} = -3Hn - \langle\sigma_{\text{eff}} v\rangle(n^2 - n^2_{\text{eq}})$

  Thermally averaged annihilation cross-section

$$\langle\sigma_{\text{eff}} v\rangle = \frac{\int_{4m^2}^{\infty} \mathrm{d}s \sqrt{s - 4m^2}\, W\, K_1(\sqrt{s}/T)}{16 m^4\, T\, K_2^2(m/T)}, \qquad W = \int \mathrm{d}\Omega\, |\mathcal{M}|^2$$

  Relic density today: $\Omega h^2 \sim \text{const.} \times \left[\int_{m/T_{\text{freezeout}}}^{\infty} \mathrm{d}x \frac{\langle\sigma_{\text{eff}} v\rangle(x)}{x^2} g_*^{1/2}\right]^{-1}$

New physics models for dark matter

- DM particles expected to be **weakly interacting**
  - → Annihilation cross-section can be computed reliably
  - → Radiative corrections small

- New symmetry predicts large spectrum of new particles

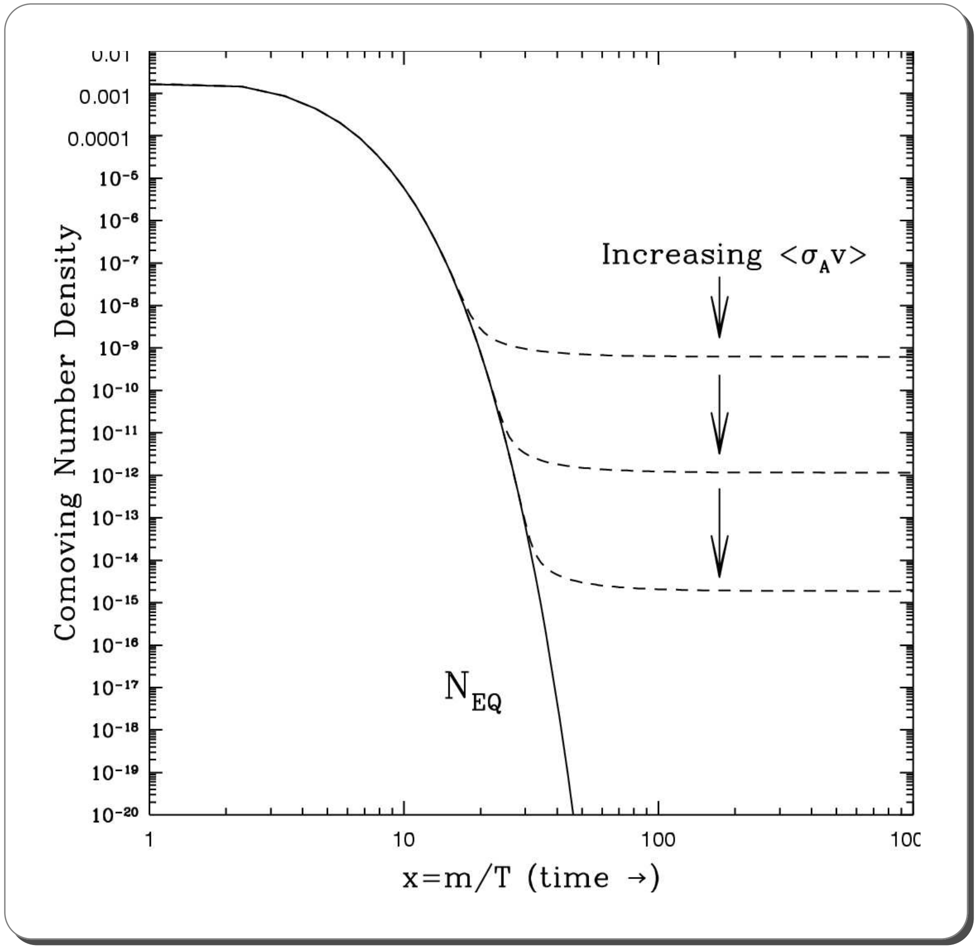
- Extra particles affect dark matter abundance
  - → Need collider data for parameters

- If mass close DM particle mass, **co-annihilation** can occur

Co-annihilation

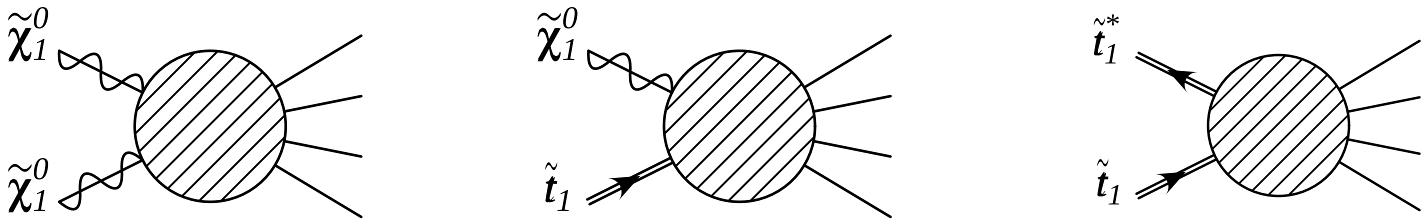
Mass of new particle $\widetilde{X}$ close to WIMP $\tilde{\chi}_1^0$

- Freeze-out of $\widetilde{X}$ and $\tilde{\chi}_1^0$ at roughly same temperature
- Annihilation in parallel (co-annihilation)
- Reduction of total dark matter density



Example:

MSSM involving
co-annihilation with scalar top



Stop-neutralino co-annihilation

- **Lightest neutralino** $\tilde{\chi}_1^0$ is good dark matter candidate in supersymmetry (for R-parity conservation)
- For bino $\tilde{\chi}_1^0$, $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to X$ is typically very small

  $\to$ Too large relic density in many SUSY scenarios
- If mass of $\tilde{t}_1$ close to mass of $\tilde{\chi}_1^0$:

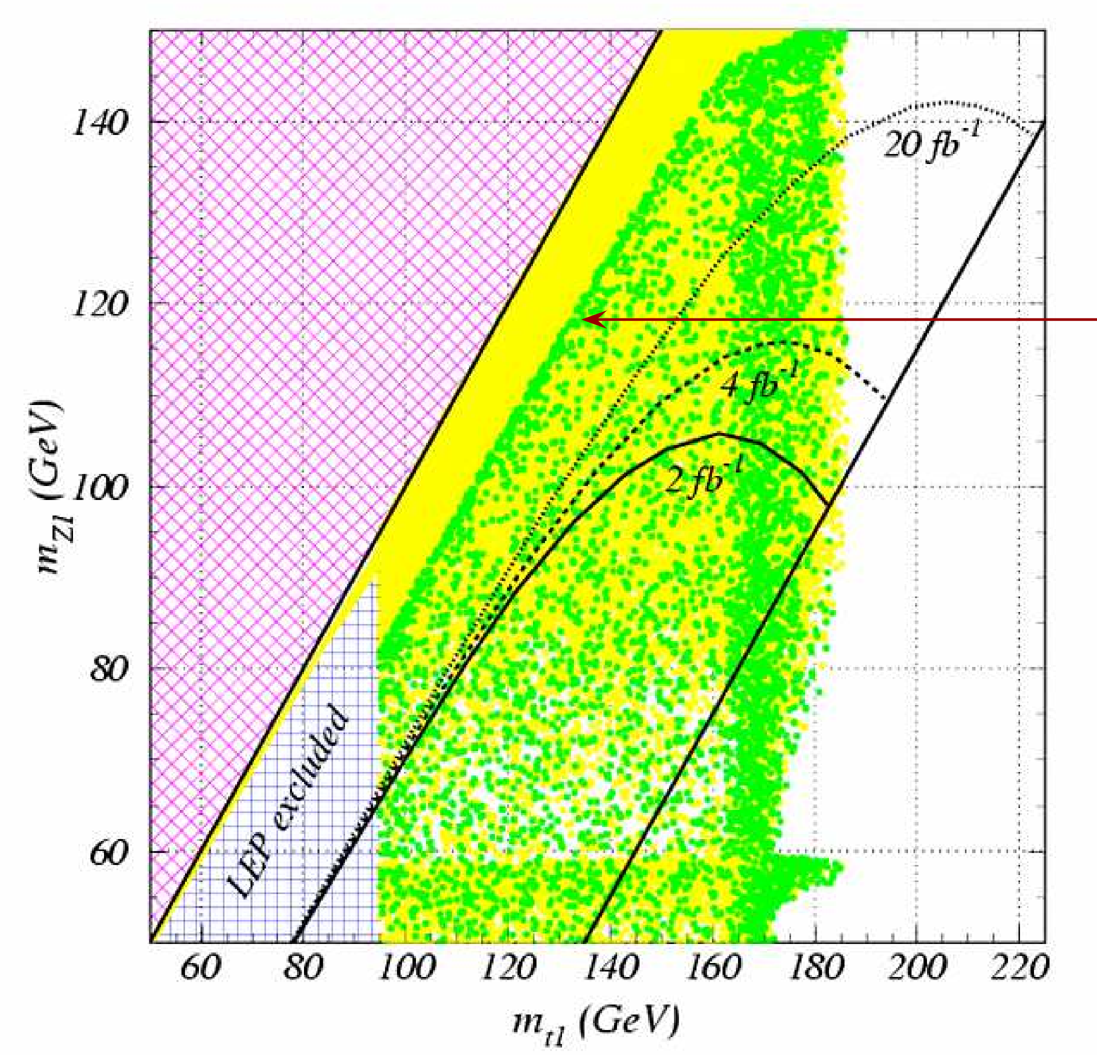
  $\to$ Stop-neutralino **co-annihilation** reduces dark matter density
- Light stop $\tilde{t}_1$ is mainly $\tilde{t}_{\mathrm{R}}$

  Assume further that $\tilde{\chi}_1^0$ is mainly bino

Typical parameter regions

Carena, Balázs, Wagner '04

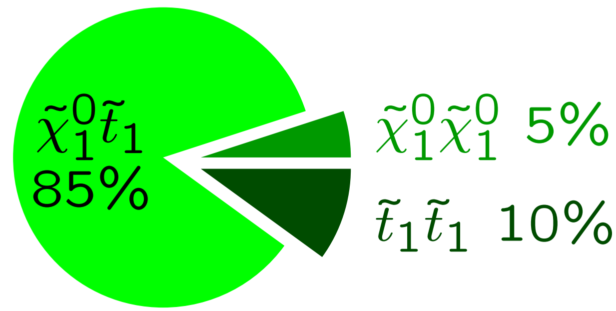
Green: Relic density consistent with WMAP



Co-annihilation for

$\Delta m \lesssim 30$ GeV

Contribution from processes:



$m_{\tilde{\chi}_1^0} = 118$ GeV

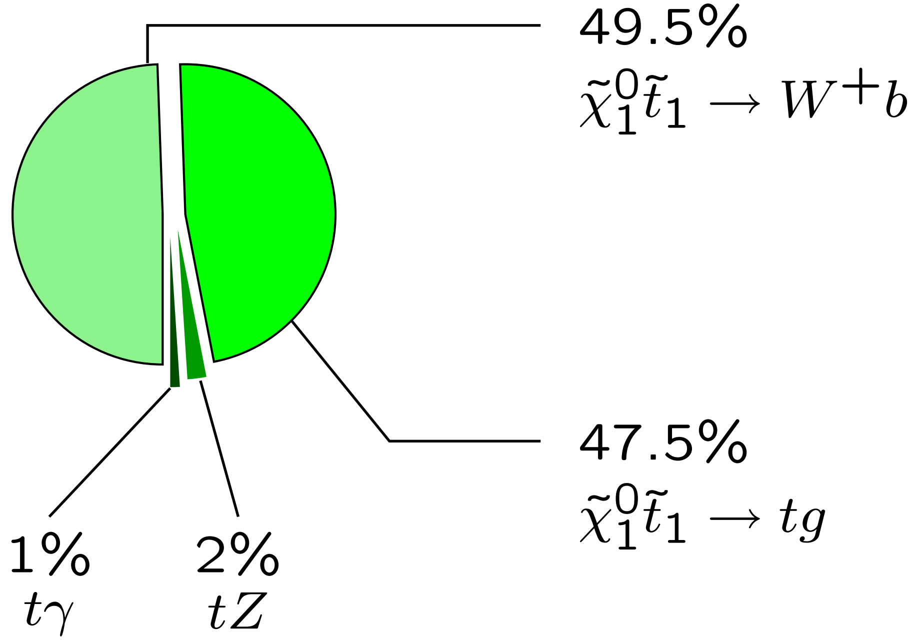
$m_{\tilde{t}_1} = 138$ GeV

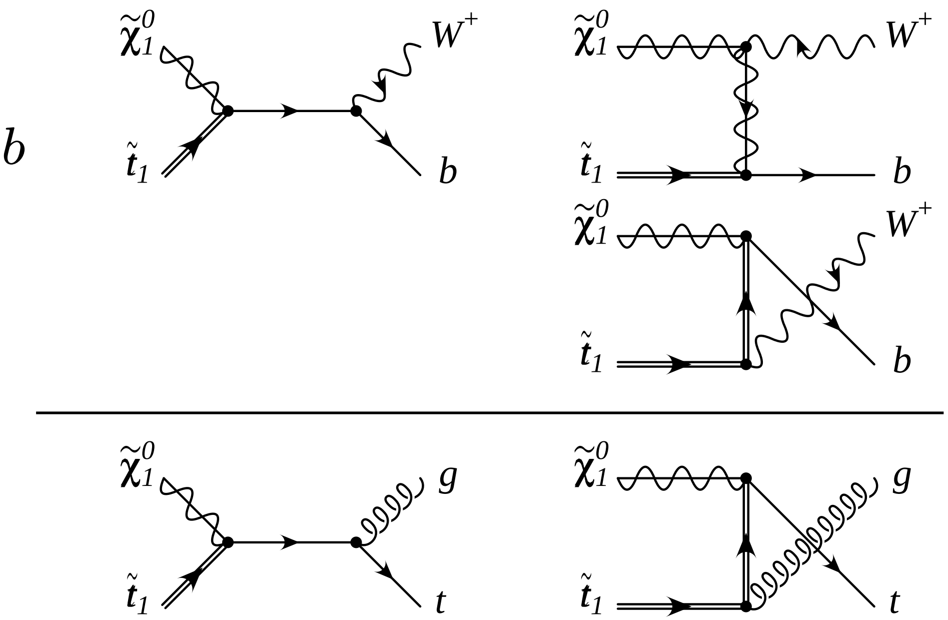
$\Omega h^2 = 0.112$

QCD corrections to $\tilde{\chi}_1^0$-$\tilde{t}_1$ annihilation

- Contrary to $\tilde{\chi}_1^0\tilde{\chi}_1^0$ annihilation, $\tilde{\chi}_1^0\tilde{t}_1$ and $\tilde{t}_1\tilde{t}_1$ annihilation receive (large) QCD corrections

- Relevant sub-processes for $\tilde{\chi}_1^0$-$\tilde{t}_1$ annihilation: ($m_{\tilde{\chi}_1^0} = 118$ GeV, $m_{\tilde{t}_1} = 138$ GeV)



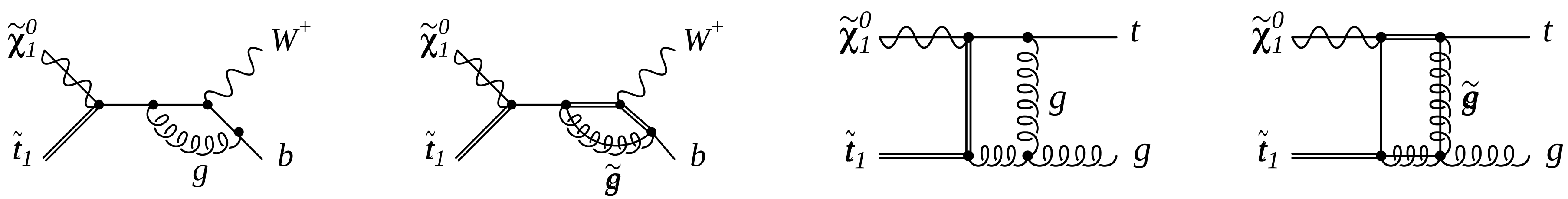


Calculation of QCD corrections

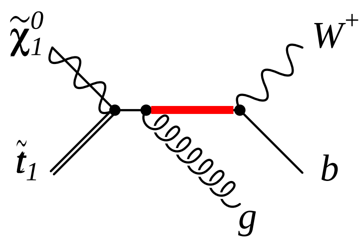
One-loop (α_s) SUSY-QCD corrections to $\tilde{\chi}_1^0 \tilde{t}_1 \to W^+ b$ and $\tilde{\chi}_1^0 \tilde{t}_1 \to tg$

Standard techniques for virtual and real contributions

Virtual corrections (examples):



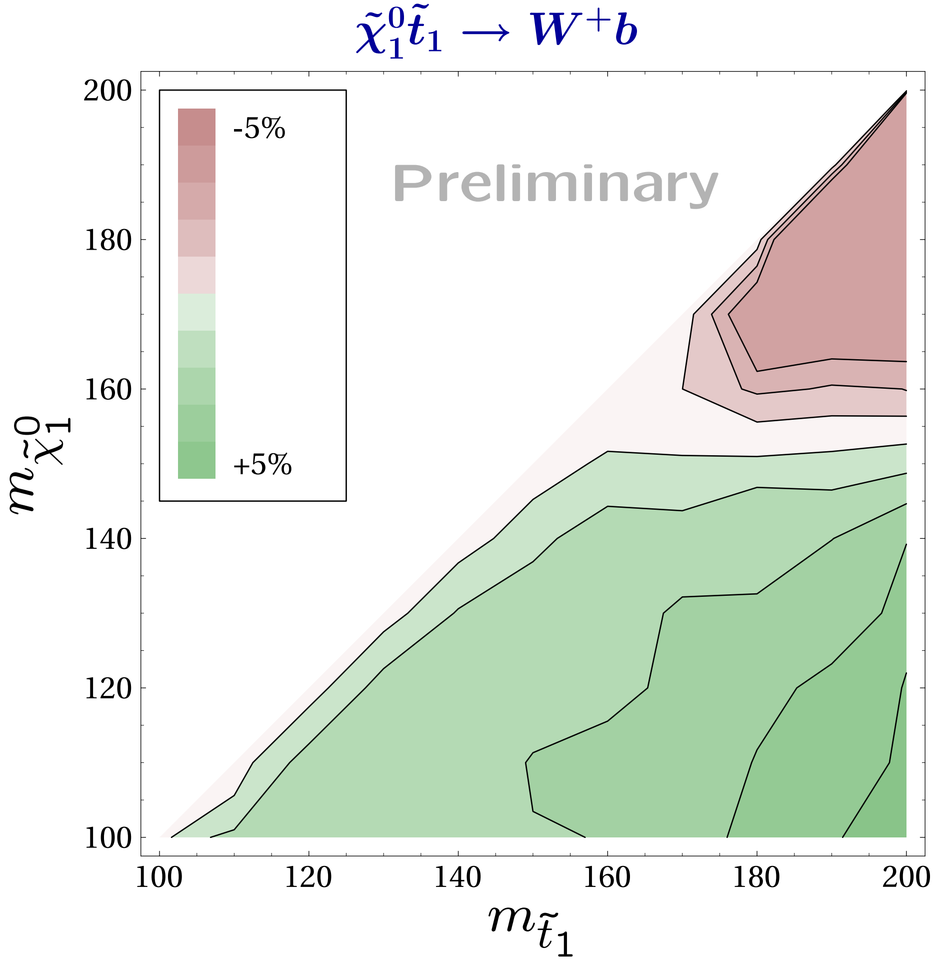
Real corrections:

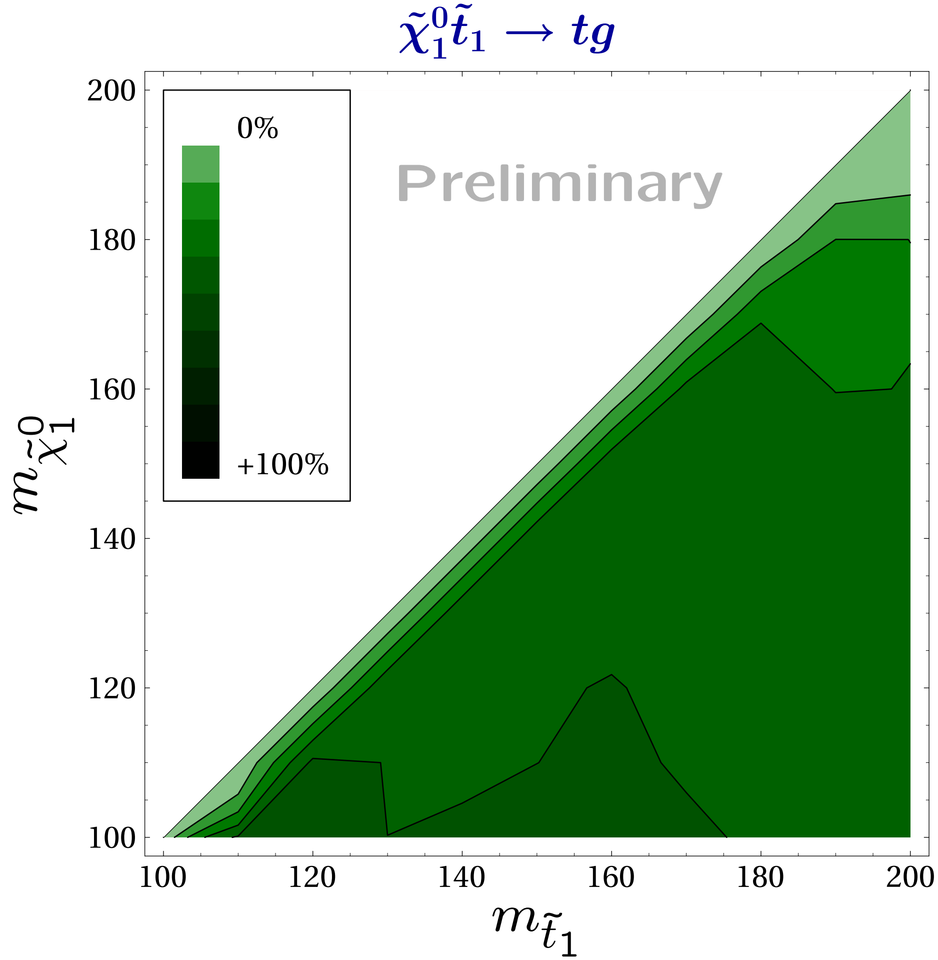


Contribution with on-shell intermediate top belongs to $\tilde{\chi}_1^0 \tilde{t}_1 \to tg$

$\to$ To be subtracted

Results for $\tilde{\chi}_1^0$-$\tilde{t}_1$ annihilation cross-section



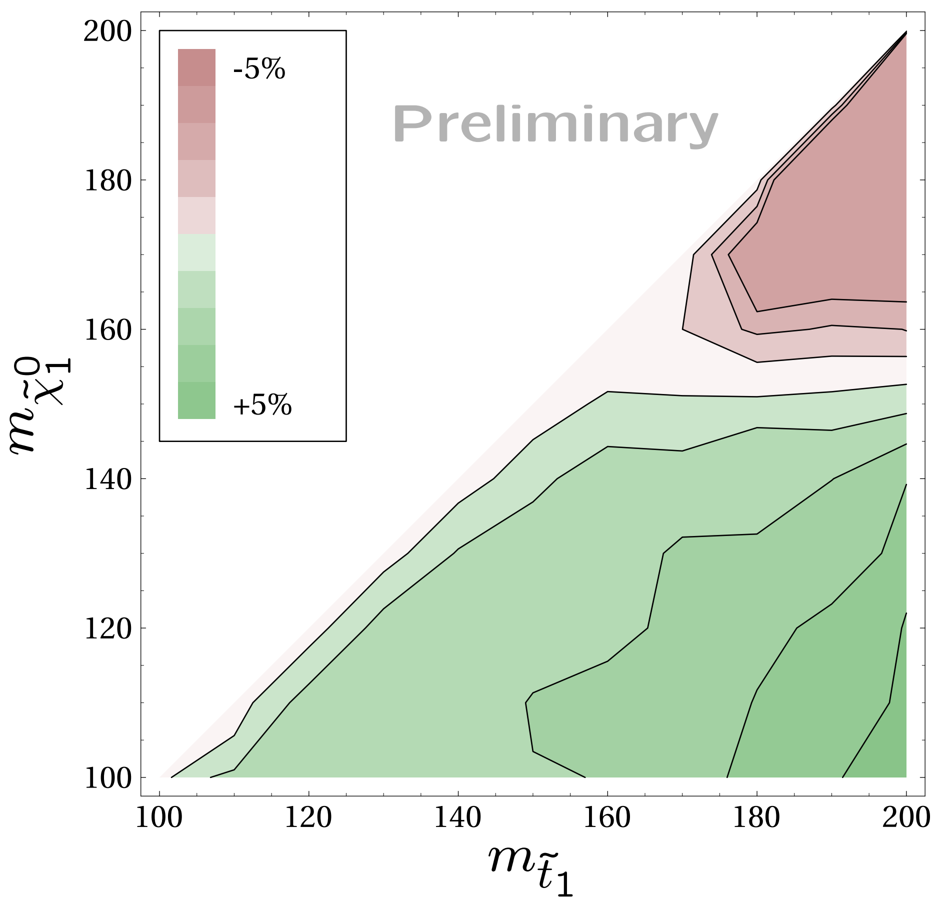


- **Few %** corrections
- Can be larger (up to $\sim 30\%$) for $m_{Wb} \approx m_{\mathrm{t}}$ due to interference effects

- Large corrections (up to $\sim 50\%$)
- Scale dependence only slightly reduced

Results for $\tilde{\chi}_1^0$-$\tilde{t}_1$ annihilation cross-section

$\tilde{\chi}_1^0 \tilde{t}_1 \to W^+ b$



- **Few %** corrections
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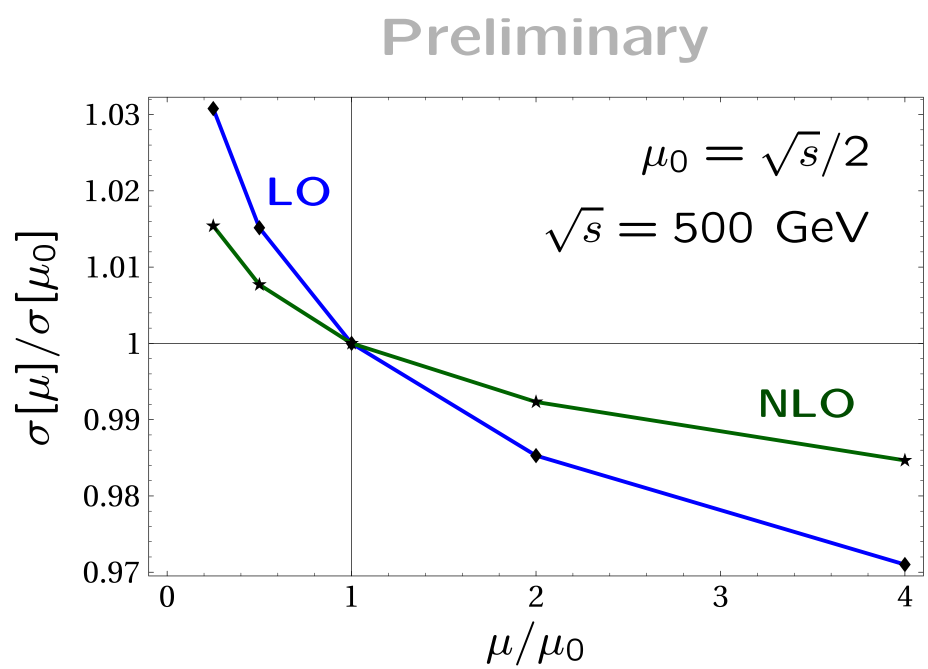
$\tilde{\chi}_1^0 \tilde{t}_1 \to tg$



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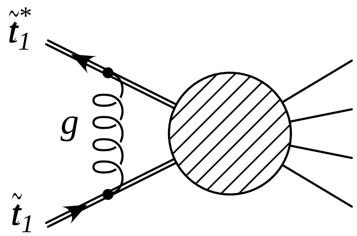
QCD corrections to $\tilde{t}_1$-$\tilde{t}_1$ annihilation

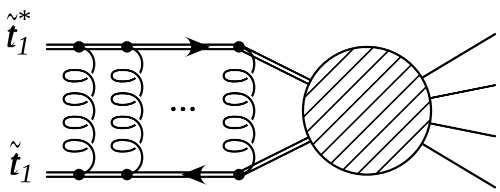
$\tilde{t}_1\tilde{t}_1$ annihilation typically contributes only 10% to relic abundance

But: During freeze-out phase stops are slowly moving
and receive large Coulomb corrections:



$$\frac{\Delta\sigma}{\sigma} \sim \frac{\alpha_s}{v} \sim \mathcal{O}(1)$$

Multiple gluon exchange gives terms $(\alpha_s/v)^n$



$\longrightarrow$ Need to be resummed

Similar to QED corrections for $\tilde{\chi}_1^0$-$\tilde{\chi}_1^\pm$ co-annihilation in focus point

Hisano, Mastumoto, Nagai, Saito, Senami '06

Coulomb correction in NRQCD

Resummed effect of Coulombic gluon corrections can be computed in non-relativistic QCD

$$\left[-\frac{\Delta}{m_{\tilde{t}_1}} + V(\boldsymbol{r})\right]\Psi(\boldsymbol{r}) = E\,\Psi(\boldsymbol{r})$$

$$V(\boldsymbol{r}) = -C_F\frac{\alpha_s}{r}, \qquad E \to E + i\Gamma$$

Correction for cross-section $\quad \dfrac{\Delta\sigma}{\sigma} = |\Psi(0)|^2$

At higher orders V receives logarithmic corrections

$$V(q^2) = -C_F\frac{4\pi\alpha_s}{q^2}\left[1 + \frac{\alpha_s}{4\pi}\left(\frac{31}{9}C_A - \frac{20}{9}T_F n_f + \beta_0 \log(\mu^2/q^2)\right) + \ldots\right]$$

Since $q = m_{\tilde{t}_1} v \ll m_{\tilde{t}_1}$, they are typically large

Coulomb correction in NRQCD

Generally, logaritms $\log(v)$ should also be resummed

Advanced methods: pNRQCD, vNRQCD

Hoang et al. '00
Beneke, Signer, Smirnov '99
Hoang, Manohar, Stewart, Teubner '01,02
Hoang '04
Penin,Piñeda,Smirnov,Steinhauser '04
Piñeda,Signer '06

Here: Simple estimate using only NRQCD with NNLO QCD potential

Schröder '99

Leading contribution comes from S-wave
$\longrightarrow$ Corresponds to 1S-stopponium bound state

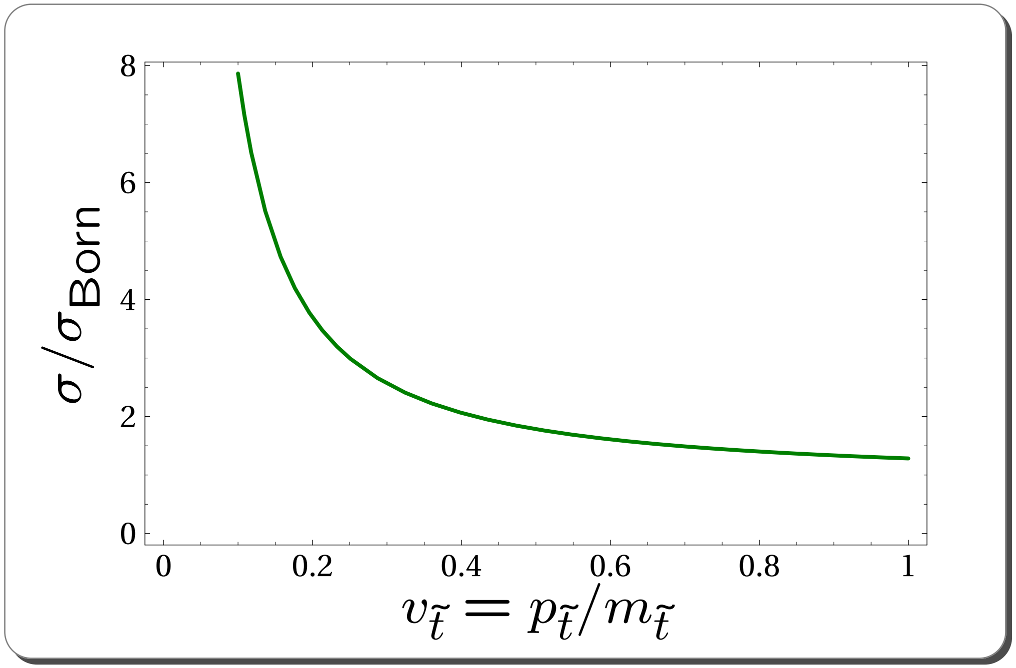
Include stopponium decay width Γ in Schrödinger equation from

$$\langle \tilde{t}_1 \tilde{t}_1 \rangle_{1\mathrm{S}} \to gg, W^+W^- \qquad \Rightarrow \qquad \Gamma \approx 5 \text{ MeV}$$

$\longrightarrow$ Very small correction

Intrinsic $\tilde{t}_1$ decay width is also very small for $m_{\tilde{t}_1} \sim \mathcal{O}(100 \text{ GeV})$

Results for $\tilde{t}_1$-$\tilde{t}_1$ annihilation cross-section



$m_{\tilde{t}_1} = 122$ GeV

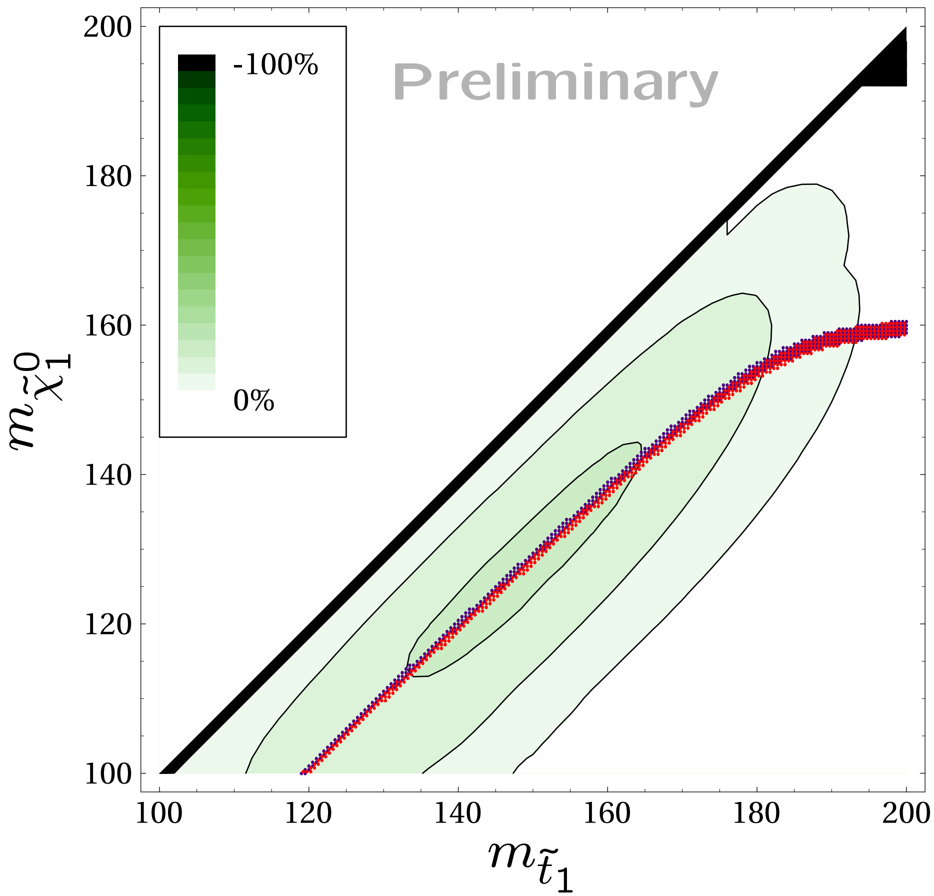
- Huge enhancement for $\beta = \sqrt{1 - 4m_{\tilde{t}_1}^2/s} \lesssim 0.4$ i.e. $\sqrt{s}/(2m_{\tilde{t}_1}) \gtrsim 1.09$
- Question when bound state effects kick in

- In freeze-out phase soft effects are cut off by temperature $T_{\text{freeze-out}} \sim \frac{1}{20} m$

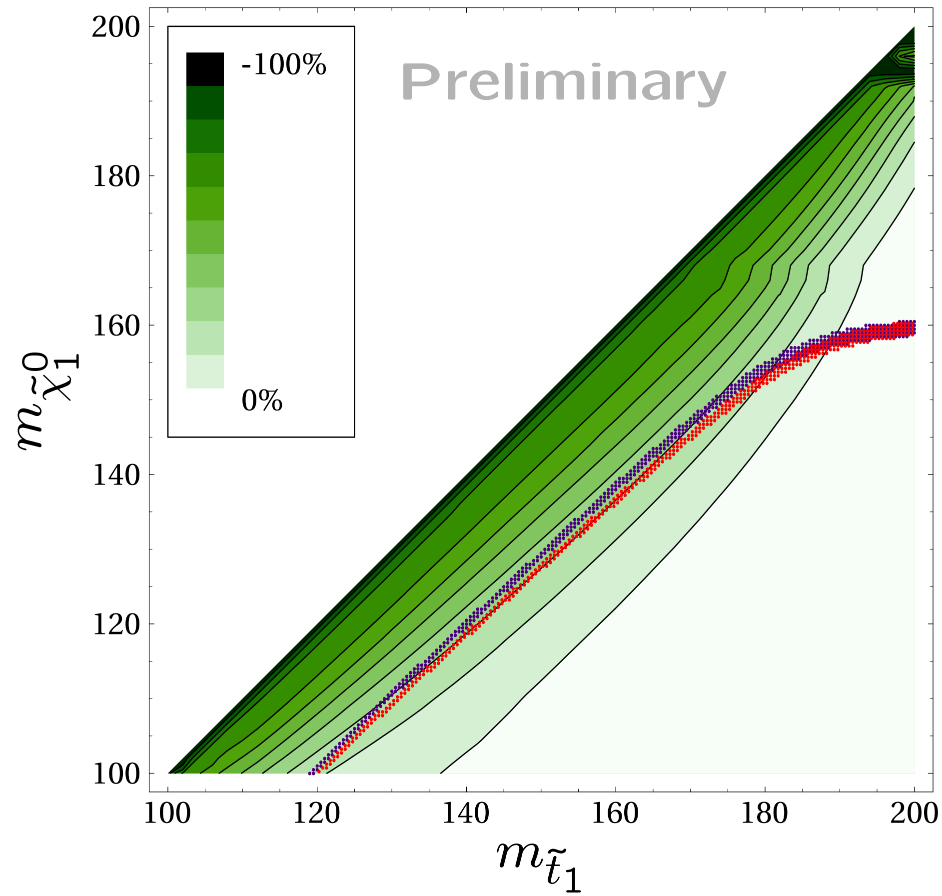
- Further improvements of theoretical predicition necessary...

Effect of radiative corrections on relic density

QCD corrections to $\tilde{\chi}_1^0\tilde{t}_1$ only



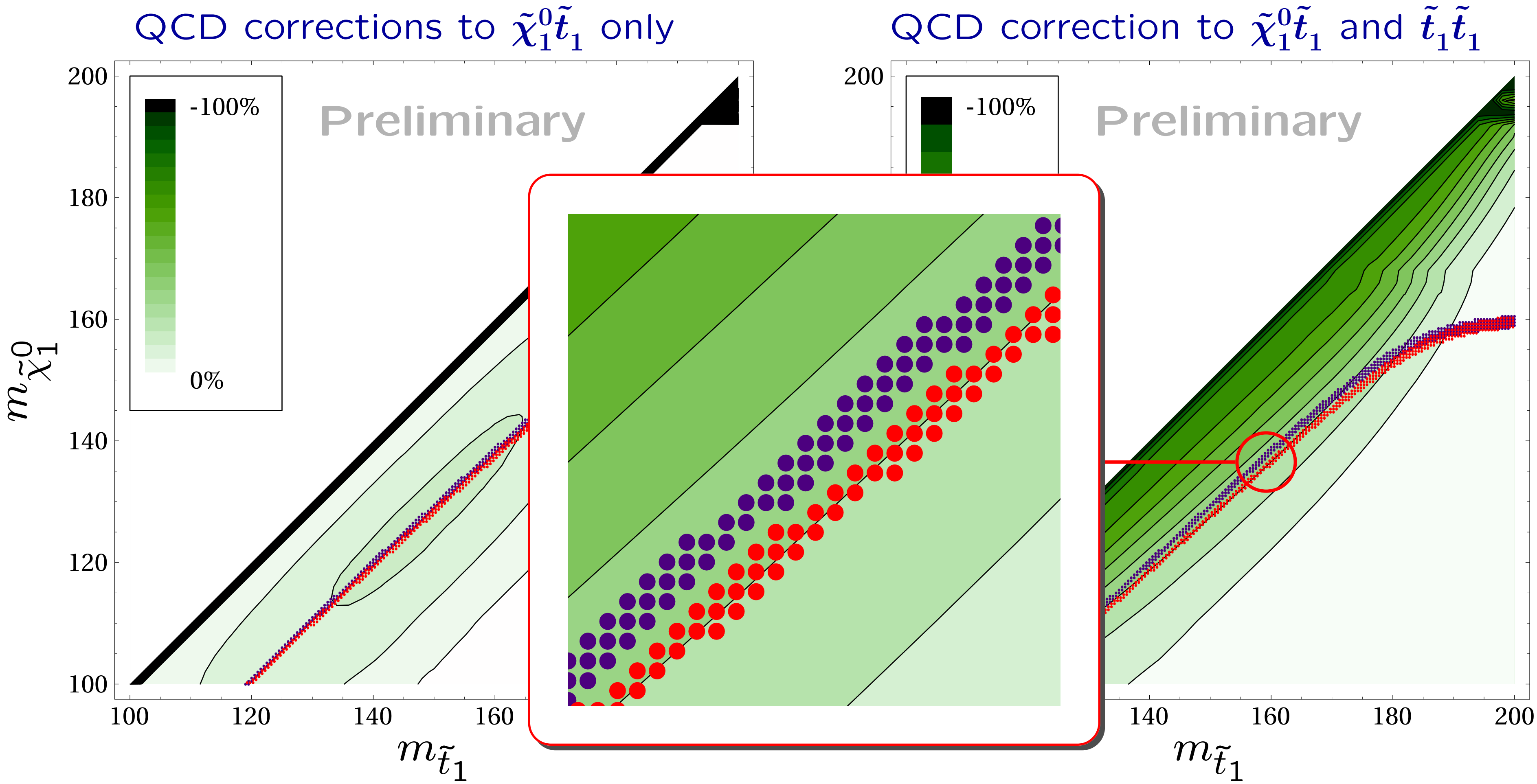
QCD correction to $\tilde{\chi}_1^0\tilde{t}_1$ and $\tilde{t}_1\tilde{t}_1$



- Medium corrections through corrections to $\tilde{\chi}_1^0\tilde{t}_1$
- Corrections larger for small masses

- Large $\tilde{t}_1\tilde{t}_1$ corrections in co-annihilation region
- WMAP preferred region shifts in parameter space

Effect of radiative corrections on relic density



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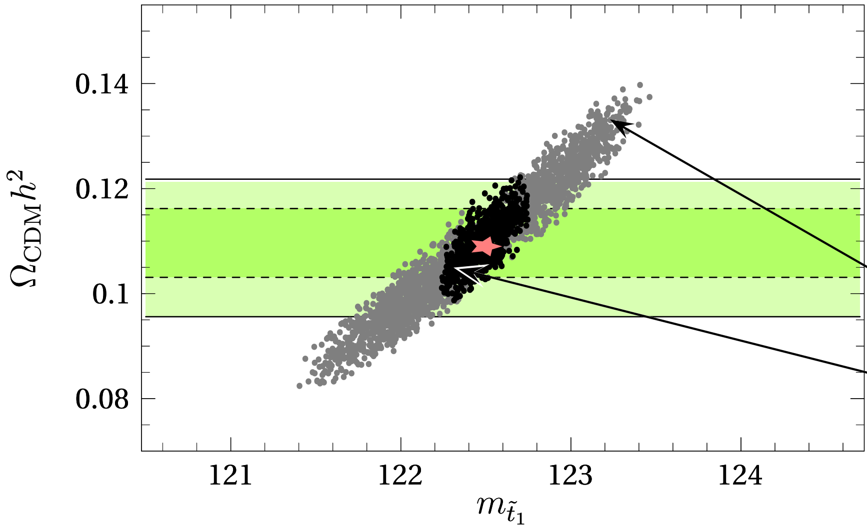
Connection to colliders

Precise measurement of $m_{\tilde{t}_1}$ and $m_{\tilde{\chi}_1^0}$ would allow to predict $\Omega_{\text{CDM}}h^2$



Benchmark scenario:
$m_{\tilde{\chi}_1^0} = 107.2$ GeV
$m_{\tilde{t}_1} = 122.5$ GeV
$\cos\theta_{\tilde{t}} = 0.0105$

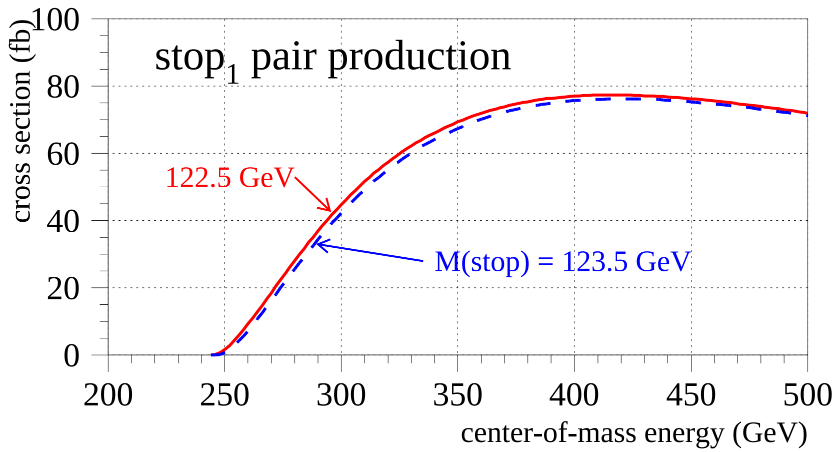
$\delta m_{\tilde{t}_1} = 1$ GeV

$\delta m_{\tilde{t}_1} = 0.3$ GeV

Carena, Freitas '06

However, large Coulombic correction introduces theoretical error

But:
Coulombic effect can be tested in $e^+e^- \to \tilde{t}_1\tilde{t}_1^*$



Conclusions

- Radiative corrections can have important impact on co-annihilation processes
- For neutralino co-annihilation, QCD corrections can reduce the predicted relic density by up to 50%
- Analysis uses only simple treatment of Coulombic QCD threshold corrections
  $\longrightarrow$ More improvements necessary