Radiative corrections to co-annihilation processes

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New physics models for dark matter

- Dark matter constitutes of **Weakly Interacting Massive Particles**, that are stable due to new symmetry (SUSY, UED, Little Higgs, ...)

- Within model, relic density can be computed

Boltzmann equation $\frac{dn}{dt} = -3Hn - \langle \sigma_{\text{eff}}v \rangle (n^2 - n_{\text{eq}}^2)$

Thermally averaged annihilation cross-section

$$\langle \sigma_{\text{eff}}v \rangle = \frac{\int_{4m^2}^{\infty} ds \sqrt{s - 4m^2} W K_1(\sqrt{s}/T)}{16m^4 T K_2^2(m/T)}$$

$$W = \int d\Omega |\mathcal{M}|^2$$

Relic density today: $\Omega h^2 \sim \text{const.} \times \left[ \int_{m/T_{\text{freezeout}}}^{\infty} dx \frac{\langle \sigma_{\text{eff}}v \rangle(x)}{x^2} g_*^{1/2} \right]^{-1}$
New physics models for dark matter

- DM particles expected to be **weakly interacting**
  - Annihilation cross-section can be computed reliably
  - Radiative corrections small

- New symmetry predicts large spectrum of new particles

- Extra particles affect dark matter abundance
  - Need collider data for parameters

- If mass close DM particle mass, **co-annihilation** can occur
Co-annihilation

Mass of new particle $\widetilde{X}$ close to WIMP $\widetilde{\chi}_1^0$

- Freeze-out of $\widetilde{X}$ and $\widetilde{\chi}_1^0$ at roughly same temperature
- Annihilation in parallel (co-annihilation)
- Reduction of total dark matter density

Example:
MSSM involving co-annihilation with scalar top

![Graph showing co-annihilation and freeze-out](image)
Stop-neutralino co-annihilation

- **Lightest neutralino** $\tilde{\chi}_1^0$ is good dark matter candidate in supersymmetry (for R-parity conservation)

- For bino $\tilde{\chi}_1^0$, $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow X$ is typically very small
  - → Too large relic density in many SUSY scenarios

- If mass of $\tilde{t}_1$ close to mass of $\tilde{\chi}_1^0$:
  - → Stop-neutralino **co-annihilation** reduces dark matter density

- Light stop $\tilde{t}_1$ is mainly $\tilde{t}_R$
  - Assume further that $\tilde{\chi}_1^0$ is mainly bino
Typical parameter regions

Carena, Balázs, Wagner '04

Green: Relic density consistent with WMAP

Co-annihilation for $\Delta m \lesssim 30$ GeV

Contribution from processes:

$\tilde{\chi}^0_1 \tilde{\tau}_1$: 85%

$\tilde{\chi}^0_1 \tilde{\chi}^0_1$: 5%

$\tilde{\tau}_1 \tilde{\tau}_1$: 10%

$m_{\tilde{\chi}^0_1} = 118$ GeV

$m_{\tilde{\tau}_1} = 138$ GeV

$\Omega h^2 = 0.112$
QCD corrections to $\tilde{\chi}_1^0 - \tilde{t}_1$ annihilation

- Contrary to $\tilde{\chi}_1^0 \tilde{\chi}_1^0$ annihilation, $\tilde{\chi}_1^0 \tilde{t}_1$ and $\tilde{t}_1 \tilde{t}_1$ annihilation receive (large) QCD corrections.

- Relevant sub-processes for $\tilde{\chi}_1^0 - \tilde{t}_1$ annihilation:
  
  
  $(m_{\tilde{\chi}_1^0} = 118 \text{ GeV}, m_{\tilde{t}_1} = 138 \text{ GeV})$

\[ 49.5\% 
\tilde{\chi}_1^0 \tilde{t}_1 \rightarrow W^+ b \]

\[ 47.5\% 
\tilde{\chi}_1^0 \tilde{t}_1 \rightarrow tg \]

\[ 1\% 
\tilde{t} \gamma \]

\[ 2\% 
\tilde{t} Z \]
Calculation of QCD corrections

One-loop ($\alpha_s$) SUSY-QCD corrections to $\tilde{\chi}_1^0 t_1 \rightarrow W^+ b$ and $\tilde{\chi}_1^0 t_1 \rightarrow tg$

Standard techniques for virtual and real contributions

Virtual corrections (examples):

Real corrections:

Contribution with on-shell intermediate top belongs to $\tilde{\chi}_1^0 t_1 \rightarrow tg$
→ To be subtracted
Results for $\tilde{\chi}_1^0 - \tilde{t}_1$ annihilation cross-section

- Few % corrections
- Can be larger (up to $\sim 30\%$) for $m_{W_b} \approx m_t$ due to interference effects

- Large corrections (up to $\sim 50\%$)
- Scale dependence only slightly reduced
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**QCD corrections to $\tilde{t}_1-\tilde{t}_1$ annihilation**

$\tilde{t}_1\tilde{t}_1$ annihilation typically contributes only 10% to relic abundance

**But:** During freeze-out phase stops are slowly moving and receive large Coulomb corrections:

$$\frac{\Delta \sigma}{\sigma} \sim \frac{\alpha_s}{v} \sim \mathcal{O}(1)$$

Multiple gluon exchange gives terms $(\alpha_s/v)^n$

→ Need to be resummed

Similar to QED corrections for $\tilde{\chi}_1^0-\tilde{\chi}_1^\pm$ co-annihilation in focus point

Hisano, Mastumoto, Nagai, Saito, Senami ’06
Coulomb correction in NRQCD

Resummed effect of Coulombic gluon corrections can be computed in non-relativistic QCD

\[ \left[ -\frac{\Delta}{m_{\bar{t}_1}} + V(r) \right] \psi(r) = E \psi(r) \]

\[ V(r) = -C_F \frac{\alpha_S}{r}, \quad E \rightarrow E + i\Gamma \]

Correction for cross-section \[ \frac{\Delta\sigma}{\sigma} = |\psi(0)|^2 \]

At higher orders \( V \) receives logarithmic corrections

\[ V(q^2) = -C_F \frac{4\pi\alpha_S}{q^2} \left[ 1 + \frac{\alpha_S}{4\pi} \left( \frac{31}{9} C_A - \frac{20}{9} T_F n_f + \beta_0 \log(\mu^2/q^2) \right) + \ldots \right] \]

Since \( q = m_{\bar{t}_1} v \ll m_{\bar{t}_1} \), they are typically large.
Coulomb correction in NRQCD

Generally, logarithms $\log(v)$ should also be resummed

Advanced methods: pNRQCD, vNRQCD

Hoang et al. '00
Beneke, Signer, Smirnov '99
Hoang, Manohar, Stewart, Teubner '01,02
Hoang '04
Penin, Piñeda, Smirnov, Steinhauser '04
Piñeda, Signer '06

Here: Simple estimate using only NRQCD with NNLO QCD potential

Schröder '99

Leading contribution comes from S-wave
→ Corresponds to $1S$-stopponium bound state

Include stopponium decay width $\Gamma$ in Schrödinger equation from

$\langle \tilde{t}_1 \tilde{t}_1 \rangle_{1S} \rightarrow gg, W^+W^- \Rightarrow \Gamma \approx 5$ MeV

→ Very small correction

Intrinsic $\tilde{t}_1$ decay width is also very small for $m_{\tilde{t}_1} \sim \mathcal{O}(100$ GeV)
Results for \(\tilde{t}_1-\tilde{t}_1\) annihilation cross-section

\[ m_{\tilde{t}_1} = 122 \text{ GeV} \]

- Huge enhancement for 
  \[ \beta = \sqrt{1 - \frac{4m_{\tilde{t}_1}^2}{s}} \lesssim 0.4 \]
  i.e. \( \sqrt{s}/(2m_{\tilde{t}_1}) \gtrsim 1.09 \)
- Question when bound state effects kick in

- In freeze-out phase soft effects are cut off by temperature
  \[ T_{\text{freeze-out}} \sim \frac{1}{20}m \]

- Further improvements of theoretical prediction necessary...
Effect of radiative corrections on relic density

- Medium corrections through corrections to $\tilde{\chi}_1^0 \tilde{t}_1$
- Corrections larger for small masses

- Large $\tilde{t}_1 \tilde{t}_1$ corrections in co-annihilation region
- WMAP preferred region shifts in parameter space
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Connection to colliders

Precise measurement of $m_{\tilde{t}_1}$ and $m_{\tilde{\chi}_1^0}$ would allow to predict $\Omega_{\text{CDM}}h^2$

Benchmark scenario:

- $m_{\tilde{\chi}_1^0} = 107.2$ GeV
- $m_{\tilde{t}_1} = 122.5$ GeV
- $\cos \theta_\ell = 0.0105$
- $\delta m_{\tilde{t}_1} = 1$ GeV
- $\delta m_{\tilde{\chi}_1^0} = 0.3$ GeV

However, large Coulombic correction introduces theoretical error

But:
Coulombic effect can be tested in $e^+ e^- \rightarrow \tilde{t}_1 \tilde{t}_1^*$
Conclusions

- Radiative corrections can have important impact on co-annihilation processes
- For neutralino co-annihilation, QCD corrections can reduce the predicted relic density by up to 50%
- Analysis uses only simple treatment of Coulombic QCD threshold corrections
  → More improvements necessary