NLO QCD corrections to $pp/p\bar{p} \rightarrow t\bar{t} + \text{jet} + X$

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Contents

1 Introduction

2 Calculation of NLO corrections

3 Numerical results

4 Conclusions
1 Introduction

Why is $pp/p\bar{p} \rightarrow t\bar{t} + \text{jet} + X$ interesting?

- Top-quark properties and dynamics still not precisely known
  → additional observables welcome, e.g.
  
  FB charge asymmetry of heavy quarks in $q\bar{q} \rightarrow t\bar{t}(\text{+jets})$
  
  origin: interference of $C$-odd / even parts initial and final states

  $q\bar{q} \rightarrow t\bar{t}$: asymmetry appears first in NLO
  Kühn, Rodrigo '98
  
  $q\bar{q} \rightarrow t\bar{t}+g$: asymmetry is LO effect
  Halzen, Hoyer, Kim '87
  
  $\leftrightarrow A_{FB}(\text{NLO})$ not feasible in near future (requires $\sigma_{\text{NNLO}}$)
  
  $\leftrightarrow A_{FB}(\text{NLO})$ can be deduced from $\sigma_{\text{NLO}}$

Situation at Tevatron: asymmetry is measurable! Bowen, S.D.Ellis, Rainwater '05

LHC: asymmetry requires preferred direction from partonic boost effects

- Important background process for Higgs search at the LHC via
  
  $pp(WW \rightarrow H) \rightarrow H + 2\text{jets}$
  
  $pp \rightarrow t\bar{t}H + X$
2 Calculation of NLO corrections

2.1 Lowest-order prediction

Some LO diagrams:

\[ gg \rightarrow t\bar{t}g: \]  
(16 diagrams)

\[ qg \rightarrow t\bar{t}q: \]  
(5 diagrams)

\[ q\bar{q} \rightarrow t\bar{t}g: \]  
(5 diagrams)

Features of the LO cross section:

- IR safety requires at least lower cut on \( p_{T,\text{jet}} \)
  \( \leftarrow \) apply jet algorithm for NLO cross section before cut on \( p_{T,\text{jet}} \)

- LO hadron cross section \( \propto \alpha_s^3 \)
  \( \leftarrow \) strong dependence on renormalization and factorization scales
Scale dependence of LO cross sections:

\[ p\bar{p} \rightarrow \bar{t}t\text{+jet+X} \quad CTEQ6L1 \]

\[ \sqrt{s} = 1.96 \text{ TeV} \]

\[ p_{T,\text{jet}} > 20 \text{GeV} \]

\[ \sigma_{\text{LO}} \quad [\text{pb}] \]

In LO: light final-state parton \( \equiv \) jet

\( \leftrightarrow \) no dependence on jet algorithm

\( \mu = \mu_{\text{ren}} = \mu_{\text{fact}} \)
2.2 Virtual corrections

# 1-loop diagrams \( \sim 350(100) \) for \( gg(q\bar{q}) \rightarrow t\bar{t}g \)

Most complicated 1-loop diagrams—pentagons of the types:

Algebraic reduction of amplitudes to standard form, e.g.

\[
\mathcal{M}_{g_ag_b \rightarrow t_i\bar{t}_jg_c} = \sum_{k=1}^{10} \sum_{l=1}^{144} F_{kl} \left( \{p_i \cdot p_j\} \right) \mathcal{C}_k \hat{\mathcal{M}}_1 \left( \{p_i\} \right)
\]

standard colour structures
\( C_1 = (T^{ca} T^{cb} T^{cc})_{ij} \), etc.

invariant functions containing loop integrals

standard spinor structures
\( \hat{\mathcal{M}}_1 = (\bar{v}_t u_{\bar{t}})(\varepsilon_a \varepsilon_b)(k_t \varepsilon_c^*) \), etc.
Two independent strategies for evaluation of loop integrals

• Calculation analogous to NLO QCD calculation for $pp \rightarrow t\bar{t}H$
  
  ◦ diagrams generated with FeynArts 1.0 Küblbeck, Böhm, Denner '90
  and reduced with in-house Mathematica routines $\rightarrow$ Fortran
  
  ◦ analytical extraction of soft / collinear singularities
    Beenakker et al. '02; S.D. '03
  
  ◦ reduction of 5-point to 4-point integrals according to Denner, S.D. '02
    $\leftrightarrow$ no (leading) inverse Gram det’s $\rightarrow$ sufficient numerical stability
  
  ◦ outlook: process will be used as further test ground for
    more sophisticated tensor reduction methods (seminumerical and/or expansion techniques)
    used at NLO EW for $e^+e^- \rightarrow 4f$
    Denner et al. '05

• Alternative calculation
  
  ◦ diagrams generated with QGRAF (Nogueira '93) and reduced with FORM $\rightarrow$ C++
  
  ◦ reduction of 5-point to 4-point integrals according to Giele, Glover '04
    $\leftrightarrow$ no (leading) inverse Gram det’s
  
  ◦ outlook: further numerical stabilization
    via expansion method suggested by Giele, Glover, Zanderighi '04
Strategy for extracting or translating IR (soft / collinear) singularities:

Idea: convert integrals $I^{(D)}$ in $D=4-2\epsilon$ dim. to 4-dim. integrals $I^{(\lambda)}$ with mass regulator $\lambda$

Procedure: consider finite and reg.-scheme-independent difference

$$\left[ I^{(D)} - I^{(D)}_{\text{sing}} \right]_{D \to 4} = \left[ I^{(\lambda)} - I^{(\lambda)}_{\text{sing}} \right]_{\lambda \to 0}$$

$$\Rightarrow I^{(D)} = I^{(D)}_{\text{sing}} + \left[ I^{(\lambda)} - I^{(\lambda)}_{\text{sing}} \right]_{\lambda \to 0} + O(\epsilon)$$

Note: mass-singular part can be universally constructed from 3-point integrals

$\leftrightarrow$ general result known explicitly

An example:

$$I^{(D)} = A_{04} \times \text{sing} + A_{43} \times \text{sing} + A_{02} \times \text{sing} + A_{42} \times \text{sing} + A_{03} \times \text{sing} \left\{ \frac{1}{\epsilon^2}, \frac{1}{\epsilon}, \ln^2 \lambda, \ln \lambda, \frac{1}{\epsilon}, \ln \lambda \right\}$$

Beenakker et al. '01

S.D. '03
2.3 Real corrections

Some diagrams with 1-parton emission

\[ gg \rightarrow t\bar{t}gg: \] (123 diagrams)

\[ gg \rightarrow t\bar{t}q\bar{q}: \] (36 diagrams)

\[ g\bar{q} \rightarrow t\bar{t}qq: \] (36 diagrams)

\[ q\bar{q} \rightarrow t\bar{t}gg: \] (36 diagrams)

Features of the calculation

• evaluation of helicity amplitudes via
  ◦ conventional (4-dimensional) spinor techniques
    automated a la Weinzierl ’05
  ◦ Berends/Giele recurrence relations

\[ |M|^2 \] checked against MADGRAPH Stelzer, Long ’94

• extraction and integration of soft / collinear singularities via
dipole subtraction formalism Catani, Seymour ’96; S.D. ’99; Phaf, Weinzierl ’01
Catani, S.D., Seymour, Trocsanyi ’02

etc.
Dipole subtraction formalism

→ process-independent treatment of singularities in real NLO corrections

worked out for

- QCD with massless partons (Catani, Seymour ’96)
- $\gamma$ radiation off massive fermions (S.D. ’99)
- QCD with massive partons (Phaf, Weinzierl ’01; Catani, S.D., Seymour, Trócsányi ’02)

basic idea: NLO correction to process with $m$ partons

$$
\sigma^{\text{NLO}} = \int_{m+1} d\sigma^{\text{real}} - d\sigma^{\text{sub}} + \int_m d\sigma^{\text{virtual}} + d\sigma^{\text{sub}}_1 + \int_0^1 dx \int_m d\sigma^{\text{fact}}(x) + \left(d\sigma^{\text{sub}}(x)\right)_+$$

conditions on $d\sigma^{\text{sub}}$:

- sum rule: $-\int_{m+1} d\sigma^{\text{sub}} + \int_m d\sigma^{\text{sub}}_1 + \int_0^1 dx \int_m \left(d\sigma^{\text{sub}}(x)\right)_+ = 0$
- asymptotics: $\sigma^{\text{sub}} \sim \sigma^{\text{real}}$ in all collinear/IR regions
2.4 Checks and status of the calculation

Summary of checks:

- **UV structure** of virtual correction
- application of different loop techniques
- soft and collinear structure in real and virtual corrections
- different methods for real-emission amplitudes, checked against MADGRAPH
- crossing symmetries
- all ingredients confirmed in second, independent calculation

Status of the calculation:

- NLO QCD calculation completed
  → first results on $\sigma_{\text{NLO}}$ and $A_{\text{FB}}(\text{NLO})$
- more numerical results including distributions in progress
- input from experimentalists welcome
  → jet definition, cuts, (in-)stability of top quarks, etc.
3 Numerical results

LO versus NLO cross section at the Tevatron and the LHC:

Jet definition:
algorithm of S.D.Ellis, Soper ’93
with $R = 1$
applied to jets other than $t$ and $\bar{t}$
Forward–backward asymmetry at the Tevatron:

S.D., Uwer, Weinzierl '07

\[ p\bar{p} \rightarrow t\bar{t} + \text{jet} + X \]
\[ \sqrt{s} = 1.96 \text{ TeV} \]
\[ p_{T,\text{jet}} > 20 \text{ GeV} \]

\[ \sigma_{\text{LO}}^\pm = \sigma_{\text{LO}}(y_t > 0) \pm \sigma_{\text{LO}}(y_t < 0) \]

\[ A_{\text{FB},\text{NLO}}^t = \frac{\sigma_{\text{LO}}^-}{\sigma_{\text{LO}}^+} \left( 1 + \frac{\delta \sigma_{\text{NLO}}^-}{\sigma_{\text{LO}}^+} - \frac{\delta \sigma_{\text{NLO}}^+}{\sigma_{\text{LO}}^-} \right) \]

\[ A_{\text{FB},\text{LO}}^t = \frac{\sigma_{\text{LO}}^-}{\sigma_{\text{LO}}^+} \]

\[ (\mu = \mu_{\text{ren}} = \mu_{\text{fact}}) \]

- \[ A_{\text{FB},\text{LO}}^t = O(\alpha_s^0) \], i.e. no dependence on \( \mu_{\text{ren}} \)
  \( \leftarrow \) mild \( \mu_{\text{fact}} \) dependence \( \ll \) theoretical uncertainty!

- \[ A_{\text{FB},\text{NLO}}^t \] depends on \( \mu_{\text{fact}} \) and \( \mu_{\text{ren}} \)
  \( \leftarrow \) asymmetry almost washed out by residual scale dependence
4 Conclusions

The process $pp/p\bar{p} \rightarrow t\bar{t} + \text{jet} + X$

- important background process for Higgs and other searches at the LHC
- interesting playground to investigate top-quark dynamics
  → measurement of FB charge asymmetry in $t\bar{t}(\text{+jets})$ already at Tevatron
  → NLO prediction for $t\bar{t}+\text{jet}$ production desirable

$pp/p\bar{p} \rightarrow t\bar{t} + \text{jet} + X$ at NLO QCD

- NLO correction stabilize LO cross sections at the Tevatron and the LHC
- FB asymmetry receives large NLO corrections
- example is important test ground for NLO methods for many-particle processes
  → methods not yet exhausted,
  more complicated applications ($2 \rightarrow 4$) feasible!