Mass effects in Higgs production at hadron colliders

Alejandro Daleo

Institut für Theoretische Physik
ETH Zürich
Work in progress on the NLO SUSY-QCD corrections to $gg \rightarrow h$

in collaboration with C. Anastasion, S. Beerli, S. Bucherer and Z. Kunszt

- introduction
- pure QCD 2 loop corrections
- toward SUSY-QCD 2 loop corrections -mostly the method-
- summary


**INTRODUCTION**

- Searches for the Higgs boson are priority at the LHC
- Gluon fusion is the dominant Higgs production mechanism at colliders
- In the SM, $gg \rightarrow h$ starts at one loop, mediated by top and bottom quarks

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**QCD corrections are large in the SM**

- $\sim 100\%$ at NLO
- $\sim 20\%$ additional NNLO (large $m_t$)
- Large $m_t$ accurate to 10-20%
CURRENT STATUS OF $gg \rightarrow h$

Standard Model:

- **full NLO corrections**
  
  Spira, Djouadi, Graudenz, Zerwas (1995)

- **NNLO corrections in the large quark mass limit**
  
  Harlander, Kilgore (2002); Anastasiou, Melnikov (2002); Ravindran, Smith, van Neerven (2003)

- **NNLO+NNLL soft gluon emission in the large quark mass limit**
  
  Catani, de Florian, Grazzini, Nason (2003)

- **Threshold-enhanced $N^3LO$ corrections in the large quark mass limit**
  

MSSM:

- **full SUSY-QCD NLO corrections in an effective theory**
  
  Harlander, Steinhauser (2004)

- **2 loop virtual squark contributions, no mixing**
  
  Anastasiou, Beerli, Bucherer, AD, Kunszt (2006), Bernreuther et al. (2006)

- **complete NLO corrections for squark contributions, no mixing**
  
  Mühlleitner, Spira (2006) ← Margarete’s talk
DO WE NEED TO GO BEYOND THIS?

- Higgs sector can be different in extensions of the SM:
  - new particles contributing to the loops, i.e. squarks, gluinos
  - different coupling structure, i.e. if $\tan \beta$ is large, bottom cont. is enhanced
  - additional, heavier, Higgs bosons

- QCD corrections expected to be also large

- is an effective theory always enough?
  - $\uparrow$ mass effects are 20 – 30% for quarks at NLO
  - $\uparrow$ corrections dominated by soft gluon emission
  - $\Rightarrow$ require a certain mass hierarchy
  - $\Rightarrow$ too many possibilities in extensions of the SM, (i.e. split SUSY, $m_\tilde{g} < m_\tilde{q}$)
WHAT DOES THAT MEAN FOR A “LOOP-FESTER”? 

For the MSSM example, this amounts to compute two loop three point diagrams with up to four different internal particles (masses)
Lot of progress in analytic calculations at two loops in the last few years

A very successful framework:

- exploit identities between loop integrals - integration by parts, Lorentz invariance - to reduce amplitudes to linear combinations of few master integrals
- use of differential equations or Mellin-Barnes representations to compute masters
- analytic continuation of results to the relevant physical regions
Some examples

- **2 → 2 partonic process in massless QCD at two loops**
  Smirnov; Smirnov, Veretin; Bern et al.; Gehrmann, Remiddi; Tausk; Anastasiou et al.

- **Two loop corrections to $e^+e^- \rightarrow 3$ jets**
  Gehrmann and Remiddi

- **Higgs boson total cross section**
  Harlander, Kilgore (2002); Anastasiou, Melnikov (2002); Ravindran, Smith, van Neerven (2003)

- **Di-lepton rapidity distribution in Drell-Yan**
  Anastasiou, Dixon, Melnikov, Petriello

- **NNLO DIS coefficient functions**
  Vermaseren, Voch, Moch

and several more...
Two loops QCD corrections to $gg \rightarrow h$
Complete analytic calculation

- amplitudes reduced to linear combinations of master integrals using integration by parts identities and Laporta’s algorithm
- complete set of master integrals computed with the method of differential equations (most were already known in the literature)
Two loops QCD corrections to $gg \rightarrow h$

Complete analytic calculation

- amplitudes reduced to linear combinations of master integrals using integration by parts identities and Laporta’s algorithm
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- analytic continuation of the master integrals above threshold
Complete analytic calculation

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Contributions mediated by a heavy quark agree with the results of Spira et al. in the analytic form derived by Harlander and Kant

First results for the scalar quark contributions, simultaneous with Aglietti et al. and Mühlleitner and Spira
SQUARK CONTRIBUTIONS AT TWO LOOPS

\[
\frac{4x C_F}{(1-x)^2} \left\{ 5 + \frac{3x^2}{(1-x)(1+x)} H(0,x) - 3H(1,x) + \frac{4x \zeta_2}{(1-x)^2} H(0,x) + \frac{8x(1+x^2) \zeta_2}{(1-x)^3(1+x)} H(0,0,0) + \frac{36x(1+x^2) \zeta_3^2}{5(1-x)^3(1+x)} + \frac{12x \zeta_3}{(1-x)^2} \right. \\
+ \frac{16x(1+x^2) \zeta_3}{(1-x)^3(1+x)} H(0,0,0) - \frac{3x(1+5x)}{(1-x)^2(1+x)} H(0,0,0) + \frac{6x}{(1-x)(1+x)} H(0,1,x) + \frac{6x}{(1-x)(1+x)} H(1,0,x) \\
+ \frac{16x}{(1-x)^2} H(0,-1,0,0) - \frac{x(-13+7x)}{(1-x)^3} H(0,0,0,0) + \frac{12x}{(1-x)^2} H(0,0,1,0) + \frac{8x}{(1-x)^2} H(0,1,0,0) \\
+ \frac{16x}{(1-x)^2} H(1,0,0,x) - \frac{16x(1+x^2)}{(1-x)^3(1+x)} H(0,-1,0,0) + \frac{32x(1+x^2)}{(1-x)^3(1+x)} H(0,0,-1,0) \\
+ \frac{2x(1+x^2)}{(1-x)^3(1+x)} H(0,0,0,0,x) - \frac{8x(1+x^2)}{(1-x)^3(1+x)} H(0,0,1,0,x) + \frac{28x(1+x^2)}{(1-x)^3(1+x)} H(0,0,0,0,x) \right\} \\
\frac{4x C_A}{(1-x)^2} \left\{ 3 - \frac{32x \zeta_2^2}{5(1-x)^2} - \frac{16x \zeta_3}{(1-x)^2} - \frac{12x \zeta_3}{(1-x)^2} H(0,x) - \frac{24x \zeta_3}{(1-x)^2} H(1,x) - \frac{2x}{(1-x)^2} H(0,0,x) + \frac{2(1-7x)x}{(1-x)^3} H(0,0,0,x) \\
+ \frac{16x}{(1-x)^2} H(1,0,0,x) - \frac{4x \zeta_2}{(1-x)^2} H(0,0,0,x) - \frac{8x \zeta_2}{(1-x)^2} H(1,0,x) - \frac{8x}{(1-x)^2} H(0,0,-1,0,x) - \frac{2x}{(1-x)^2} H(0,0,0,0,x) \\
- \frac{16x}{(1-x)^2} H(1,0,-1,0,x) + \frac{8x}{(1-x)^2} H(1,0,0,0,x) \right\}
\]
Would this approach work for the SUSY-QCD corrections?

- many scales present, the reduction gets cumbersome -as in 1 loop multileg-
- need to solve multiply coupled differential equations -like in Bhabha scattering-
- with more scales present, master integrals are combinations of complicated functions -multidimensional HPLs for example-
- time consuming method

maybe not hopeless but certainly a formidable task . . .
Two general and fully automated methods to deal with multiloop integrals in a numerical way

- **sector decomposition** Binoth, Heinrich
  - successfully implemented in several cross section calculations
    Anastasiou, Melnikov, Petriello; Melnikov, Petriello
  - not possible to handle thresholds

- **numerical integration of Mellin-Barnes representations** Anastasiou, AD; Czakon
  - works fine both in Euclidean and physical regions
  - has problems in most loop integrals with internal masses
  - most probably cannot deal with thresholds

None of these two, by itself, could do \( gg \rightarrow h \) at two loops
introduced by Soper and Nagy, to treat numerically the integration over Feynman parameters of loop integrals

the key idea: enforce the $i\delta$ prescription to avoid the zeros of the denominator function inside the Feynman parameter integration volume

schematically

\[
\int_0^1 dx_1 \cdots dx_n \frac{F(\vec{x}, \epsilon)}{G(\vec{x}, M^2_i, s_{kl}) - i\delta} \rightarrow \int_C dz_1 \cdots dz_n \frac{F(\vec{z}, \epsilon)}{G(\vec{z}, M^2_i, s_{kl})} \alpha
\]

used by Binoth et al. to evaluate numerically infrared finite loop integrals \(\Leftarrow\) Thomas’ talk
Starting from a Feynman parametrization

\[ I = C(\epsilon) \int_0^1 dx_1 \cdots dx_n \frac{\mathcal{F}(\vec{x}, \epsilon)}{\left[ \mathcal{G}(\vec{x}, M_i^2, s_{kl}) - i\delta \right]^{\alpha+n_L\epsilon}} \]

and using sector decomposition to extract the overlapping singularities, we get a sum of terms looking like

\[ I_s = C(\epsilon) \int_0^1 dx_1 \cdots dx_n x_1^{-\alpha_1+\beta_1\epsilon} \cdots x_n^{-\alpha_n+\beta_n\epsilon} \mathcal{F}_s(\vec{x}, \epsilon) \]

we deform the region of integration into the complex plane

\[ \int_0^1 \left( \prod_{j=1}^n dx_j x_j^{-\alpha_j+\beta_j\epsilon} \right) \frac{\mathcal{F}_s(\vec{x}, \epsilon)}{\left[ \mathcal{G}_s(\vec{x}, M_i^2, s_{kl}) - i\delta \right]^{\alpha+n_L\epsilon}} = \int_C \left( \prod_{j=1}^n dz_j z_j^{-\alpha_j+\beta_j\epsilon} \right) \frac{\mathcal{F}_s(\vec{z}, \epsilon)}{\left[ \mathcal{G}_s(\vec{z}, M_i^2, s_{kl}) \right]^{\alpha+n_L\epsilon}} . \]

where the contour is parametrized by

\[ z_i = x_i - i\lambda x_i(1-x_i) \frac{\partial \mathcal{G}_s}{\partial x_i} . \]
The choice for the contour deformation gives, for small values of $\lambda$:

$$
G_s(\bar{x}) - i \delta \rightarrow G_s(\bar{z}) = G_s(\bar{x}) - i \lambda \sum_i x_i (1 - x_i) \left( \frac{\partial G_s}{\partial x_i} \right)^2 + \mathcal{O}(\lambda^2)
$$

now we change variables back to the $x_i$

$$
I_s = C(\epsilon) \int_0^1 \prod_{j=1}^n dx_j z_j^{-\alpha_j + \beta_j \epsilon} \mathcal{J}(\bar{x} \rightarrow \bar{z}) \mathcal{L}(\bar{z}(\bar{x}), \epsilon)
$$

$$
= C(\epsilon) \int_0^1 \prod_{j=1}^n dx_j x_j^{-\alpha_j + \beta_j \epsilon} \left( \frac{z_j}{x_j} \right)^{-\alpha_j + \beta_j \epsilon} \mathcal{J}(\bar{x} \rightarrow \bar{z}) \mathcal{L}(\bar{z}(\bar{x}), \epsilon)
$$

and the last step is to extract the singularities at $x_i = 0$

$$
\int_0^1 dx x^{-n+\epsilon} f(x) = \int_0^1 dx x^\epsilon \frac{f(x) - \sum_{k=0}^{n-1} x^k \frac{f^{(k)}(0)}{k!}}{x^n} + \sum_{k=0}^{n-1} \frac{f^{(k)}(0)}{k!(k + 1 - n + \epsilon)}
$$
IMPLEMENTATION

- 3 independent implementations in **Maple** and **Mathematica**
  - sector decomposition
  - contour deformation
  - generation of **Fortran** and C++ code for the numerical integration
- use of **Cuba** library for numerical integration, **Divonne** algorithm well suited for these integrands
- proper treatment of higher order singularities that involve derivatives in the expansion in plus distributions

\[
\int_0^1 dx x^{\epsilon} f(x) = \int_0^1 dx x^{\epsilon} \frac{f(x) - f(0) - x f'(0)}{x^2} + \frac{f(0)}{\epsilon - 1} + \frac{f'(0)}{\epsilon}
\]
IMPLEMENTATION

- 3 independent implementations in MAPLE and MATHEMATICA
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  - generation of FORTRAN and C++ code for the numerical integration
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- proper treatment of higher order singularities that involve derivatives in the expansion in plus distributions
- control of the size of the deformation during integration

\[ G_s(x) - i \delta \to G_s(z) = G_s(x) - i \lambda \sum_i x_i (1 - x_i) \left( \frac{\partial G_s}{\partial x_i} \right)^2 + O(\lambda^2) \]
IMPLEMENTATION

- 3 independent implementations in MAPLE and MATHEMATICA
  - sector decomposition
  - contour deformation
  - generation of FORTRAN and C++ code for the numerical integration
- use of CUBA library for numerical integration, DIVONNE algorithm well suited for these integrands
- proper treatment of higher order singularities that involve derivatives in the expansion in plus distributions
- control of the size of the deformation during integration
- magnitude of the deformation ($\lambda$) important but not crucial

$$z_i = x_i - i\lambda x_i (1-x_i) \frac{\partial G_s}{\partial x_i}.$$
IMPLEMENTATION

3 independent implementations in 

Mathematica

sector decomposition

contour deformation

generation of FORTRAN and C++ code for the numerical integration

use of CUBA library for numerical integration,

DIVONNE algorithm well suited for these integrands

proper treatment of higher order singularities that involve derivatives in the expansion in plus distributions

control of the size of the deformation during integration

magnitude of the deformation (\[ \tau \]) important but not crucial

\[
\int_0^1 dx \left( f(x) + f(0) x^2 + f(0) \right) \]

\[
\int_0^1 dx \left( f(x) + f(0) x^2 + f(0) \right) = \int_0^1 dx \left( f(x) + f(0) x^2 + f(0) \right)
\]

\[
\tau = 1.05
\]

\[
\tau = 1.75
\]
A TEST: QCD CORRECTIONS TO $gg \rightarrow h$

(whole amplitude evaluated with similar precision)

$$\text{Re}(c_0)$$

$\Delta_{\text{Analytic}} \%$

$\Delta_{\text{Numeric} - \text{Analytic}}$

$$= C \left( \frac{c_1}{\epsilon} + c_0 \right)$$
SUSY-QCD corrections can be split into three classes
- QCD corrections to quark and squark mediated $gg \rightarrow h$ known
- SUSY-QCD corrections to squark loops involving the scalar quartic vertex
- SUSY-QCD corrections involving gluinos

Diagrams with gluinos involve either three or four internal masses

No infrared singularities present, no need to do sector decomposition
analytic results for 2 loop quark contributions to $gg \rightarrow h$
(first independent check of results by Spira et al.)

novel results for the 2 loop squark contributions

purely numerical calculation of both quark and squark 2 loop corrections using Sector Decomposition + Contour Deformation

numerical results for the gluino contributions

numerical convergence much better than for the pure QCD diagrams
STATUS OF NLO SUSY CALCULATION

analytic results for 2 loop quark contributions to $gg \rightarrow h$ (first independent check of results by Spira et al.)

novel results for the 2 loop squark contributions

purely numerical calculation of both quark and squark 2 loop corrections using SECTOR + CONTOUR DEFORMATION

numerical results for the gluino contributions

numerical convergence much better than for the pure QCD diagrams

(checking corrections containing gluinos)

working on squarks contributions involving the quartic scalar vertex

\begin{align*}
\text{Re}(c_0) & \quad 0 \quad 0.5 \quad 1 \quad 1.5 \quad 2 \\
\text{Im}(c_0) & \quad 0 \quad 0.5 \quad 1 \quad 1.5 \quad 2 \\
\tau & \quad 0 \quad 0.5 \quad 1 \quad 1.5 \quad 2
\end{align*}

(integration errors are better than 1%)
analytic results for 2 loop quark contributions to $gg \rightarrow h$

(first independent check of results by Spira et al.)

novel results for the 2 loop squark contributions

purely numerical calculation of both quark and squark 2 loop corrections using Sector Decomposition + Contour Deformation

numerical results for the gluino contributions

numerical convergence much better than for the pure QCD diagrams

checking corrections containing gluinos

working on squarks contributions involving the quartic scalar vertex
moving steadily towards SUSY-QCD NLO corrections to $gg \rightarrow h$
- 2 loop contributions with gluinos evaluated numerically
- working on the pure squark pieces

the combination of sector decomposition and contour deformation is a new technique with great potential

might open the door to many calculations
Lazopoulous, Melnikov and Petriello ⇐ Kirill’s talk in this workshop

it is easy to automatize and can reduce notably the (human) time needed for a calculation